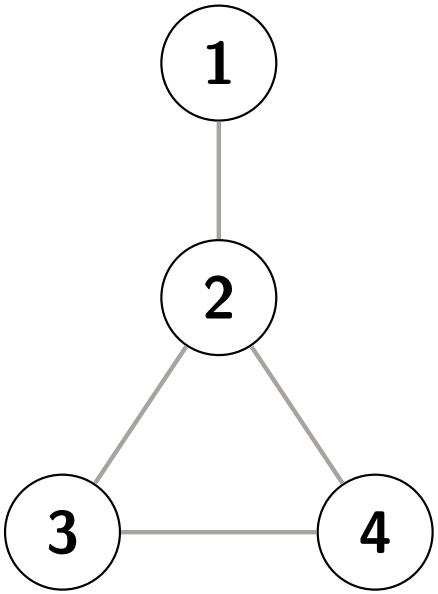
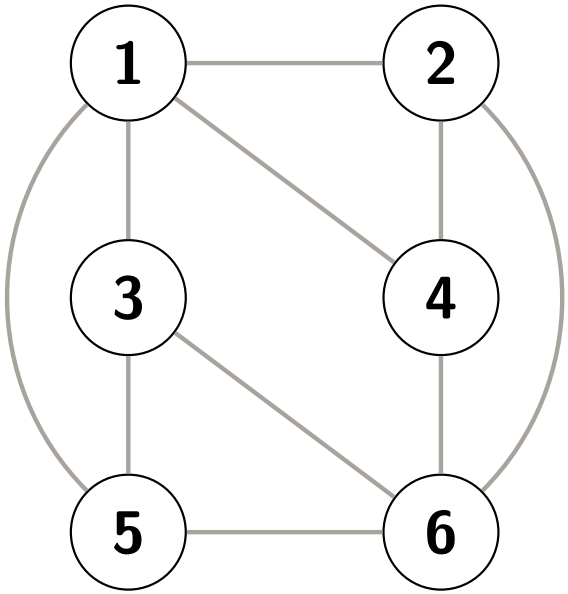
When is Finding Little Graphs Inside Big Graphs Hard?

Ciaran McCreesh, Patrick Prosser and James Trimble

University of Glasgow

Non-Induced Subgraph Isomorphism





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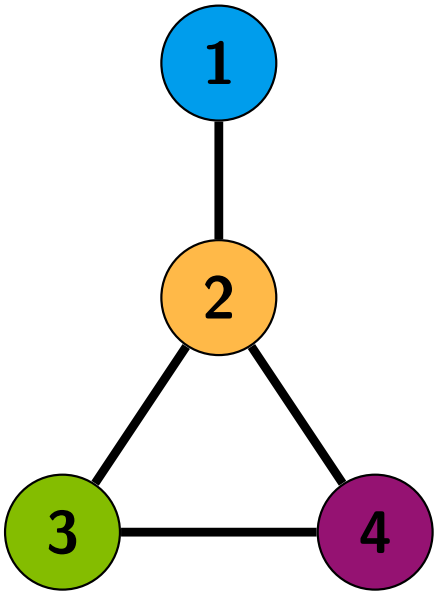
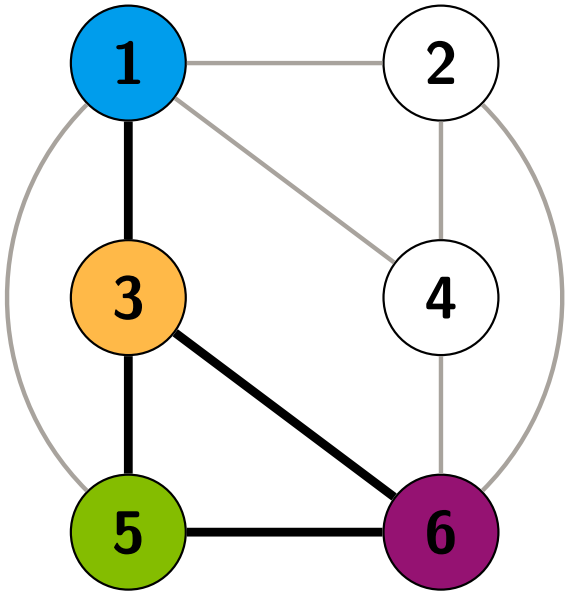
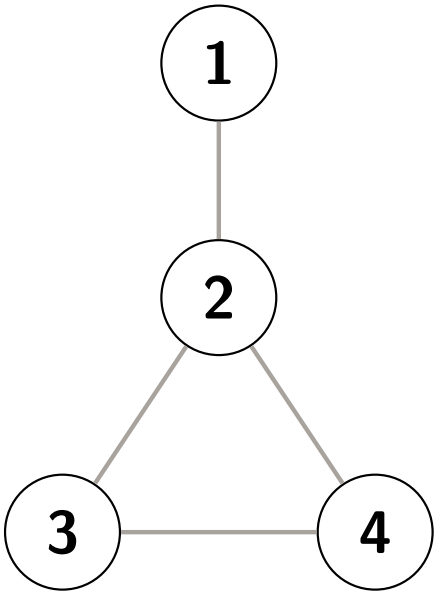
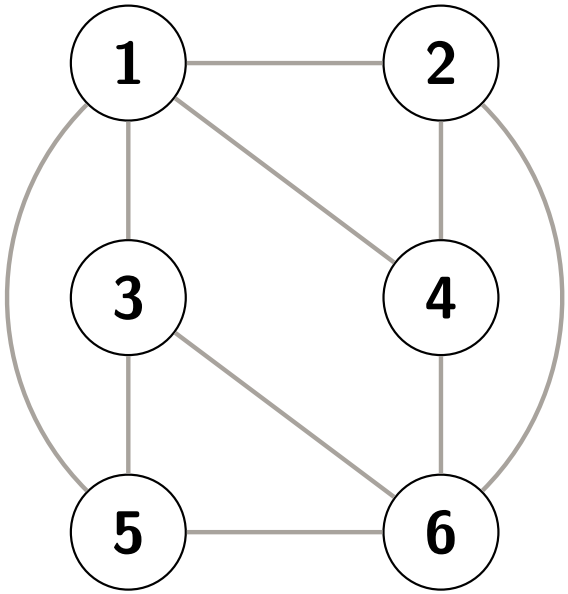




Induced Subgraph Isomorphism





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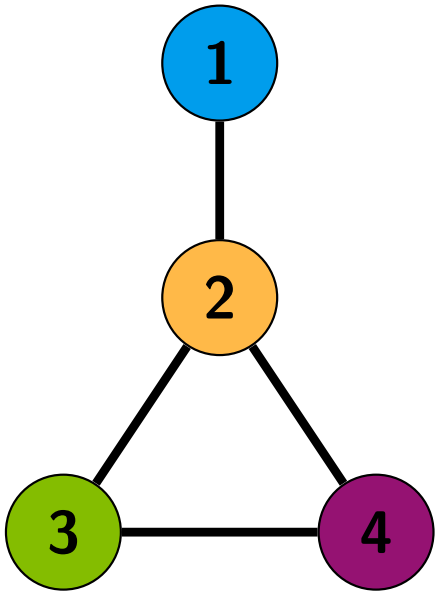
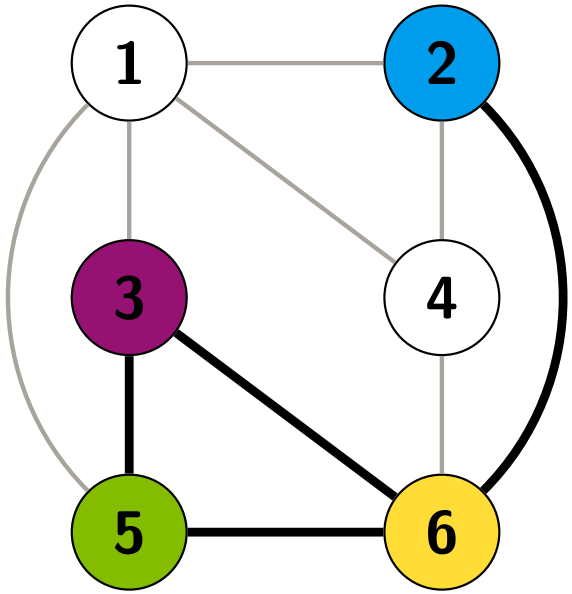




Benchmarking

- Based upon chemical and computer vision datasets, we can handle patterns with 1,000 vertices and targets with 10,000 vertices.
- Do these results reflect the worst case, or are they too optimistic?
- Can we create "hard" benchmark instances?

Randomly Selected Subgraphs

- Start with a random target graph.
- Pick vertices at random to make a pattern.
- Shuffle the numbering.

Randomly Selected Subgraphs

- Start with a random target graph.
- Pick vertices at random to make a pattern.
- Shuffle the numbering.
- These instances will always be satisfiable!

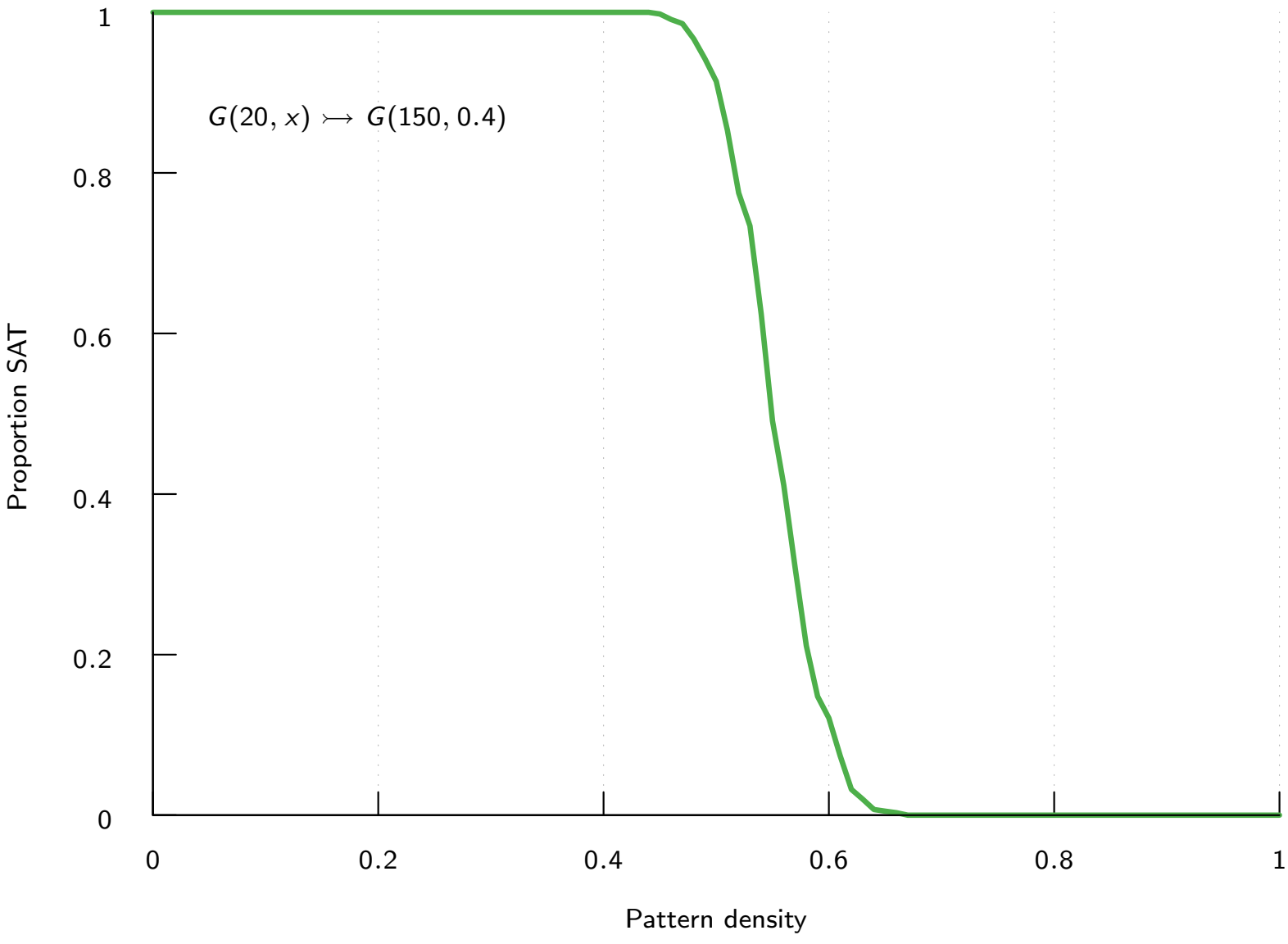
Independently Random Subgraphs

- Make a random target graph.
- Independently, make a random pattern graph.

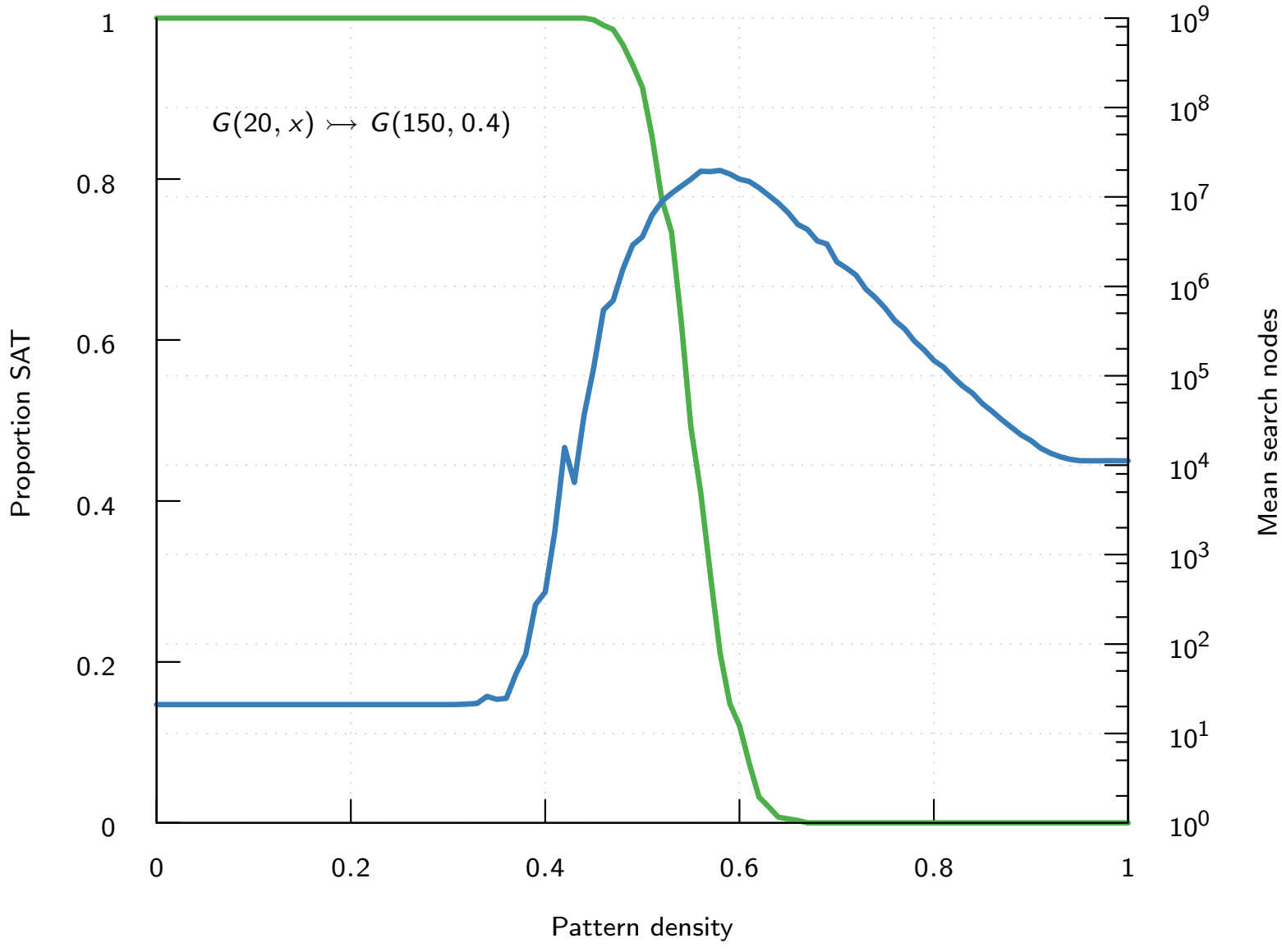
Independently Random Subgraphs

- Make a random target graph.
- Independently, make a random pattern graph.
- Will these instances ever be satisfiable?

A Phase Transition



A Phase Transition



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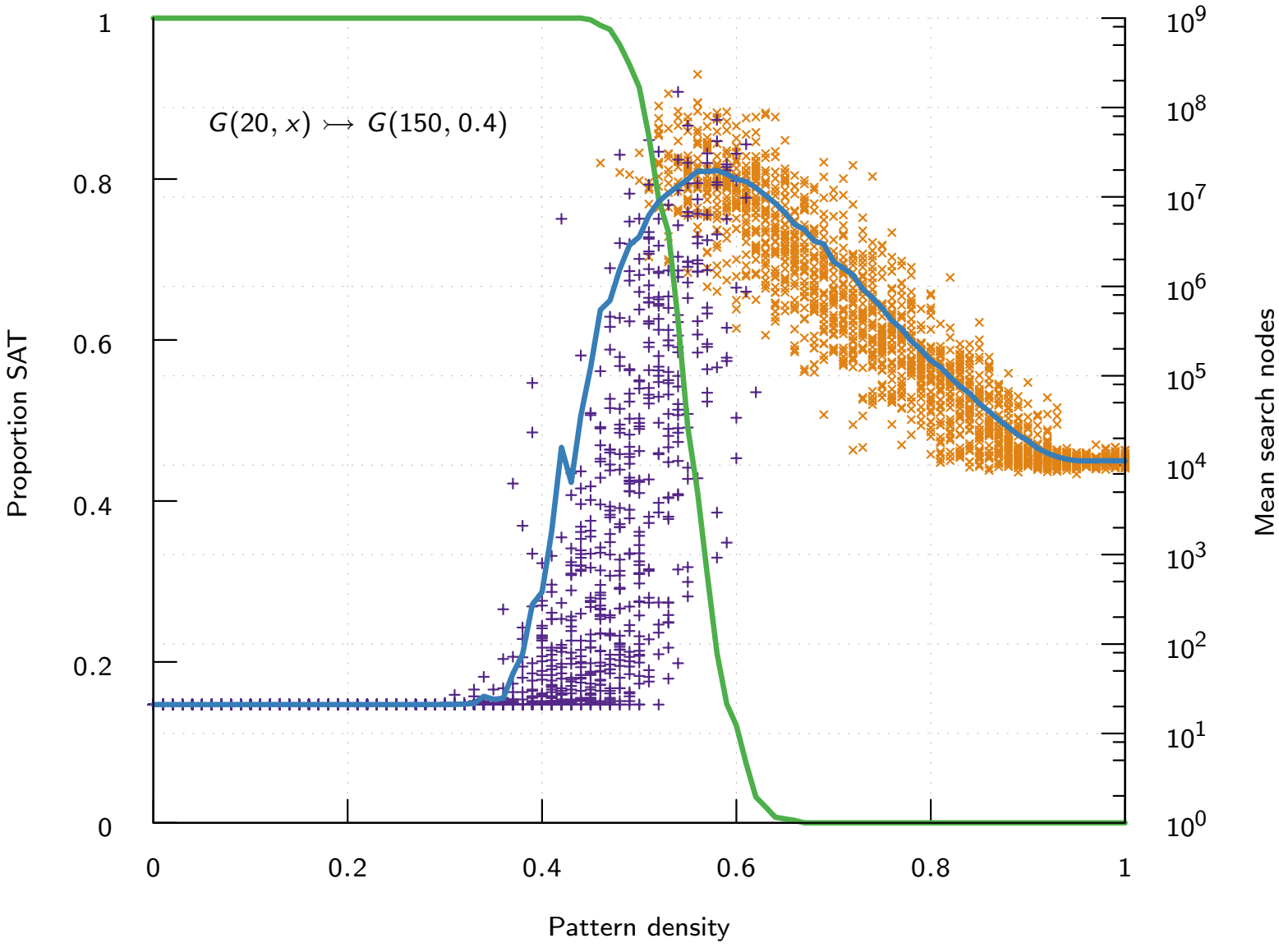
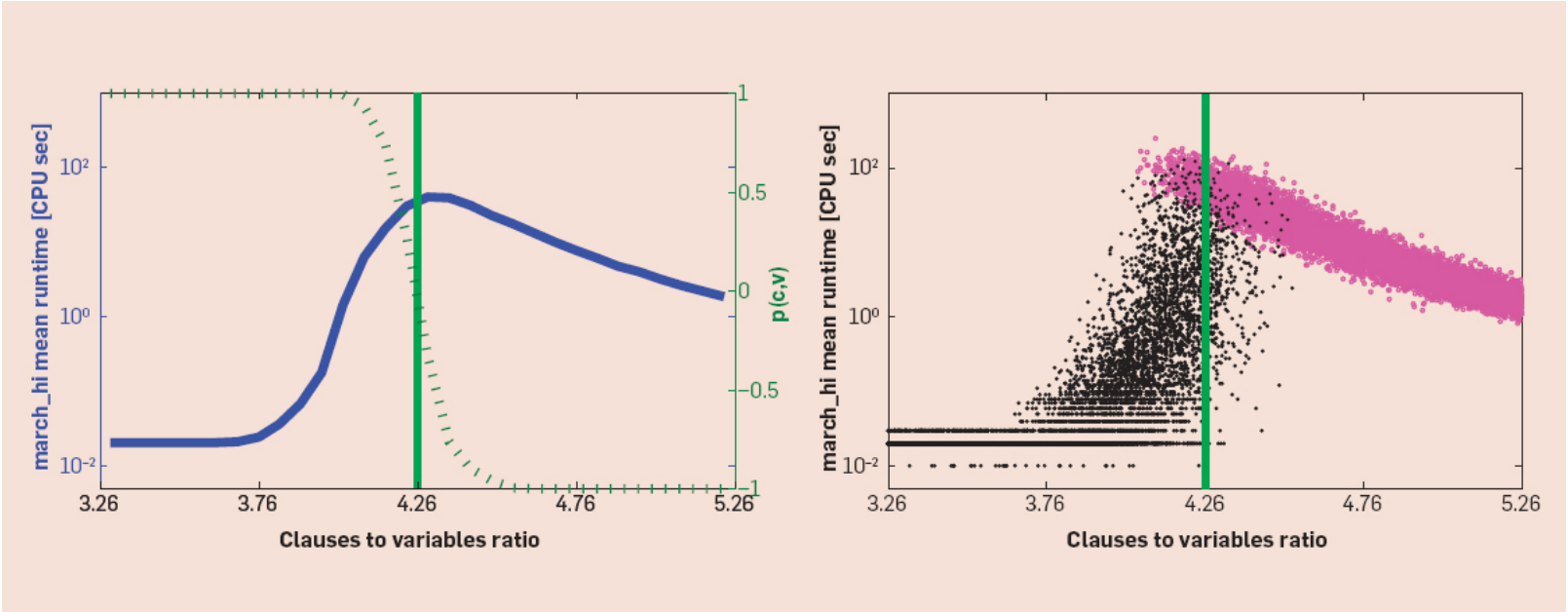


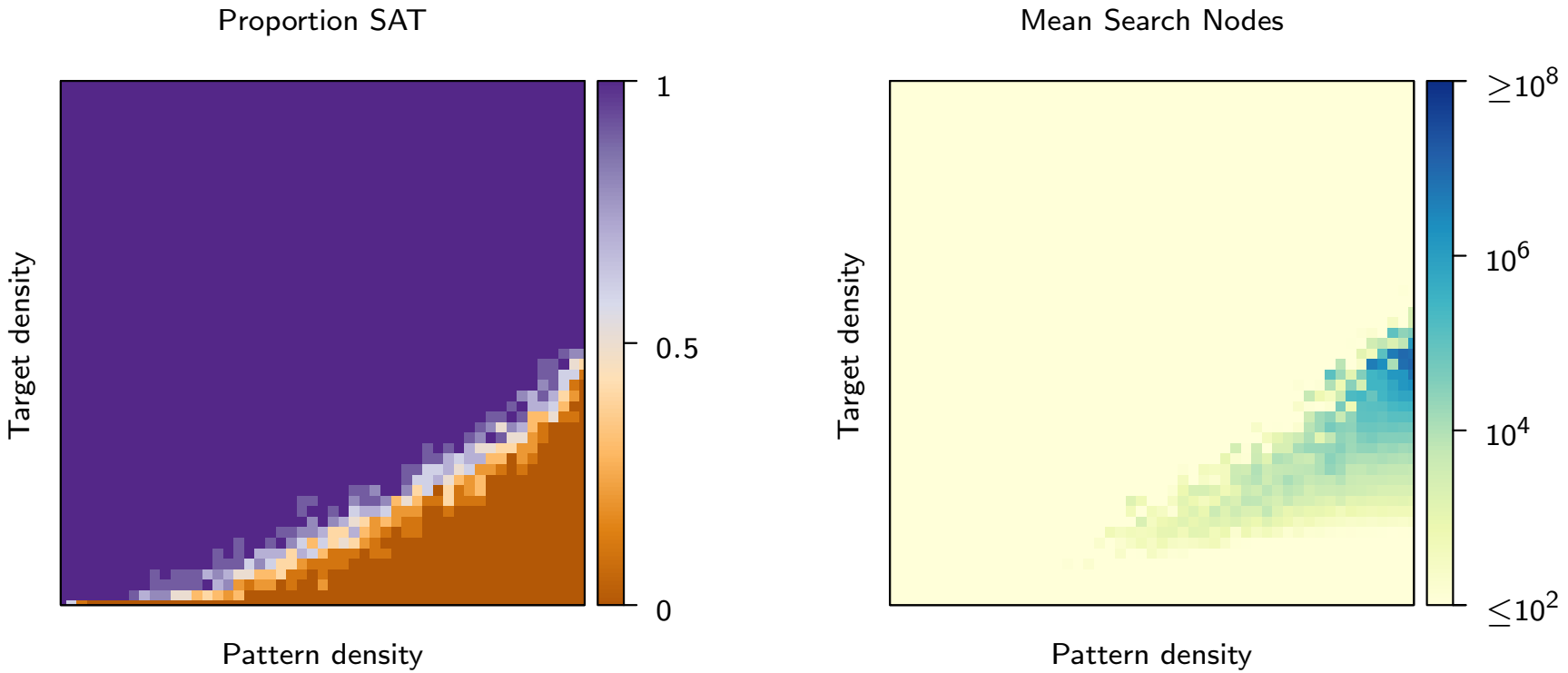
This Looks Familiar...



Understanding the Empirical Hardness of NP-Complete Problems.
Kevin Leyton-Brown, Holger H. Hoos, Frank Hutter, Lin Xu.
Communications of the ACM, Vol. 57 No. 5, Pages 98-107

2D Phase Transitions

$$G(10, x) \rightarrowtail G(150, y)$$



2D Phase Transitions

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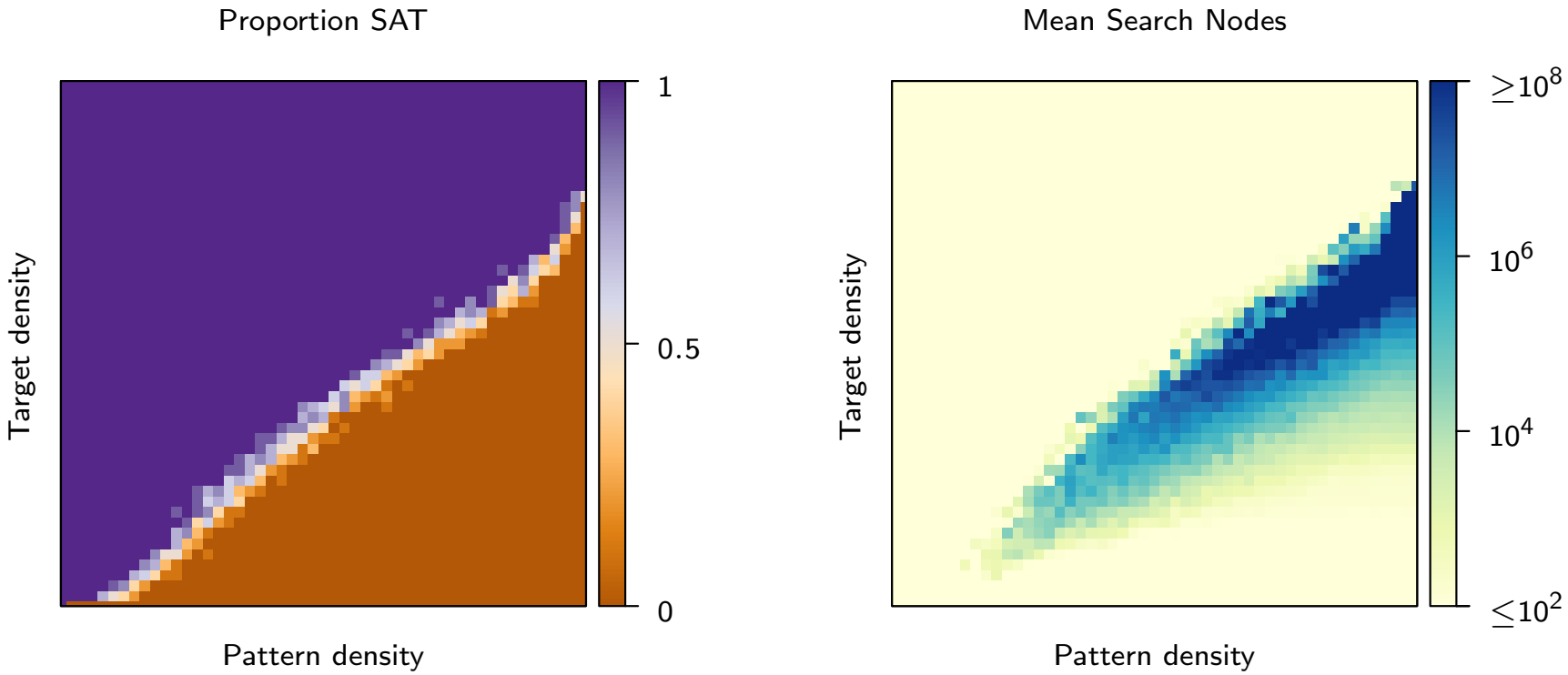
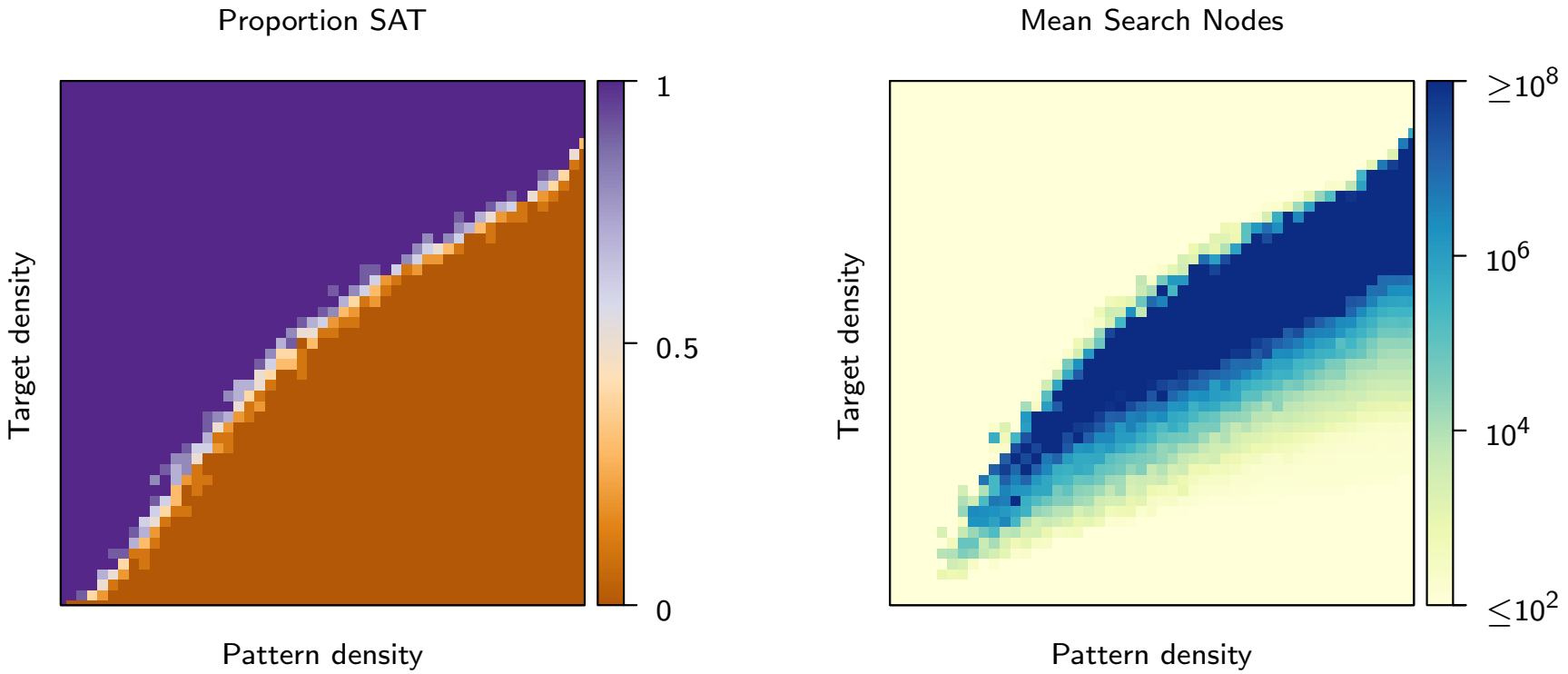
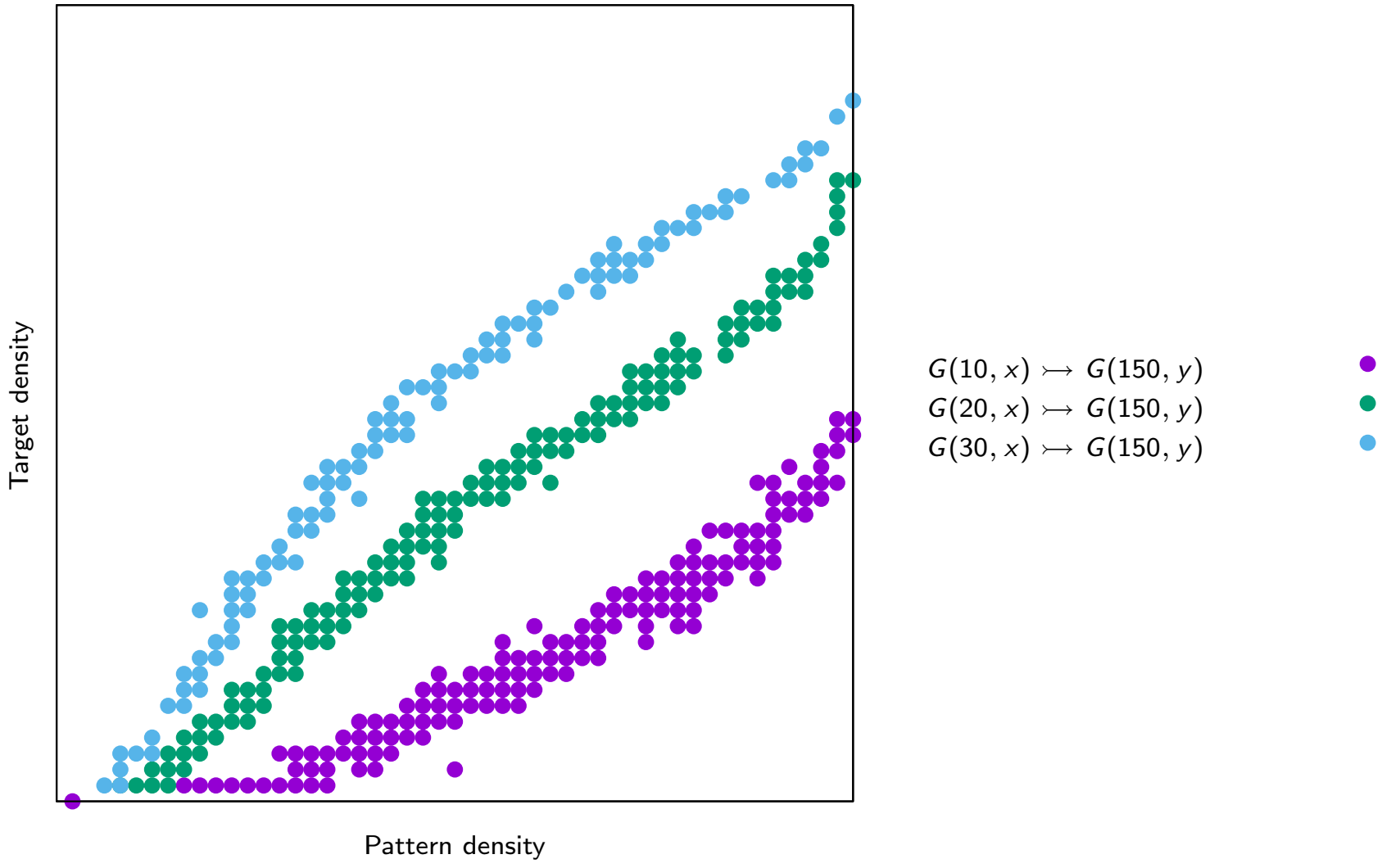


2D Phase Transitions

$$G(30, x) \rightarrowtail G(150, y)$$



3D Phase Transitions?



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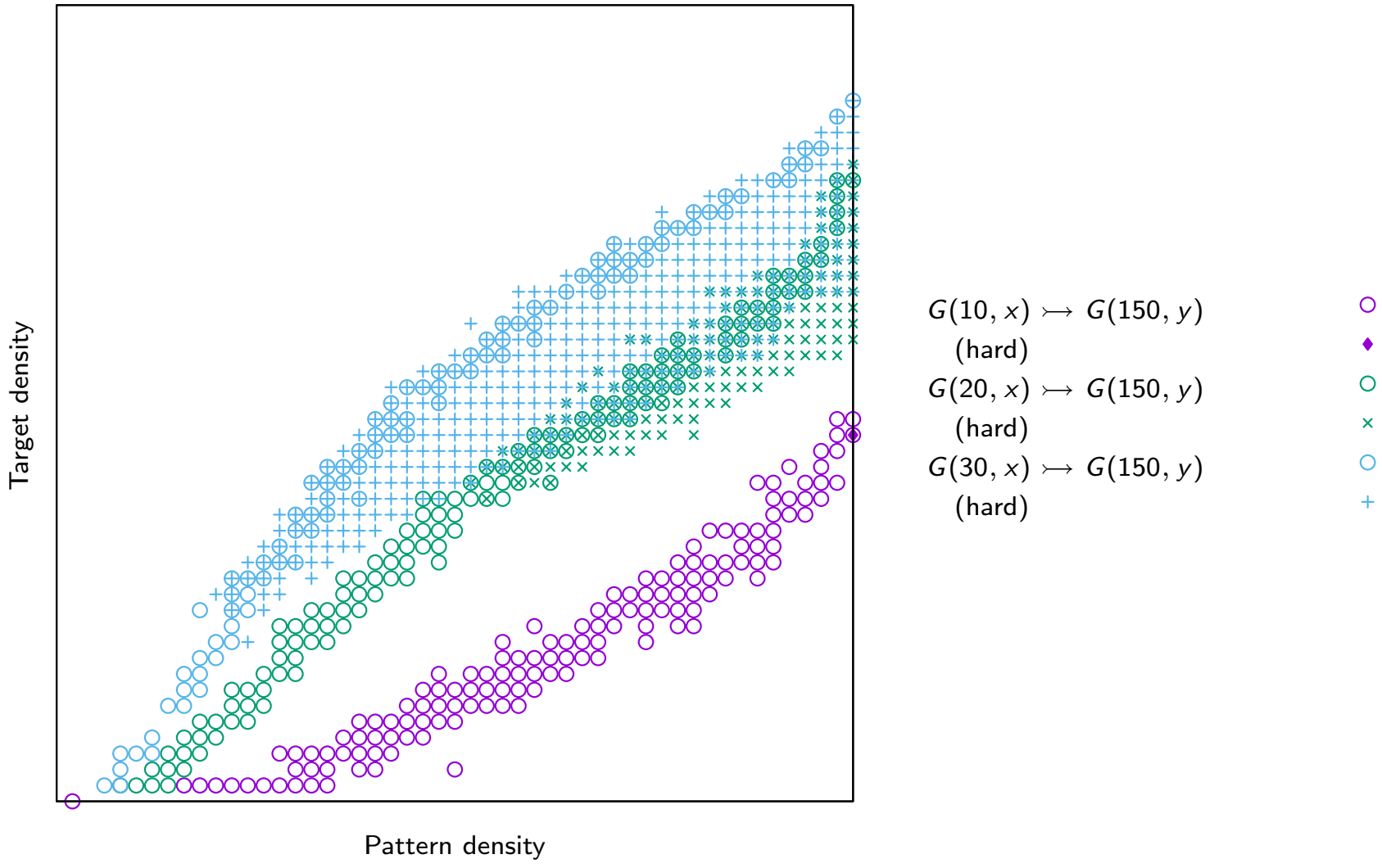
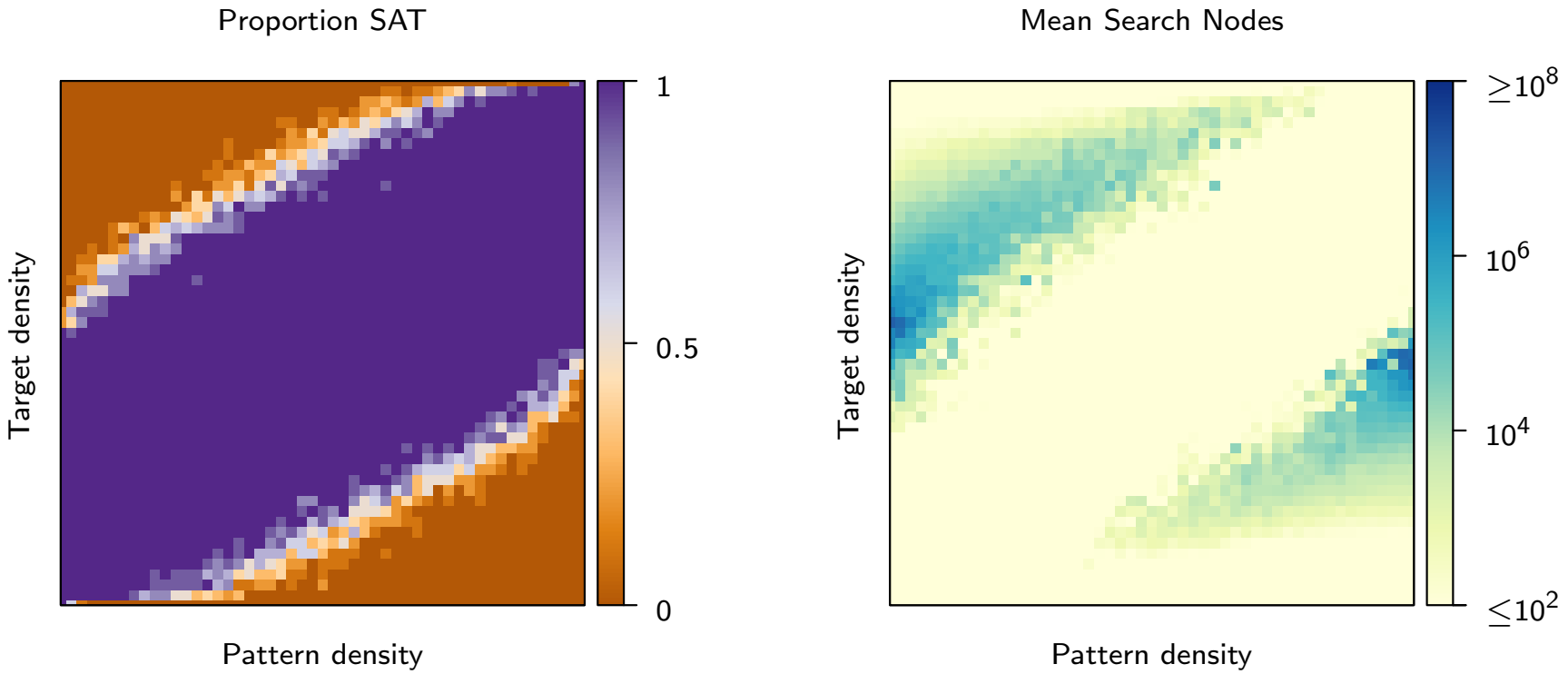


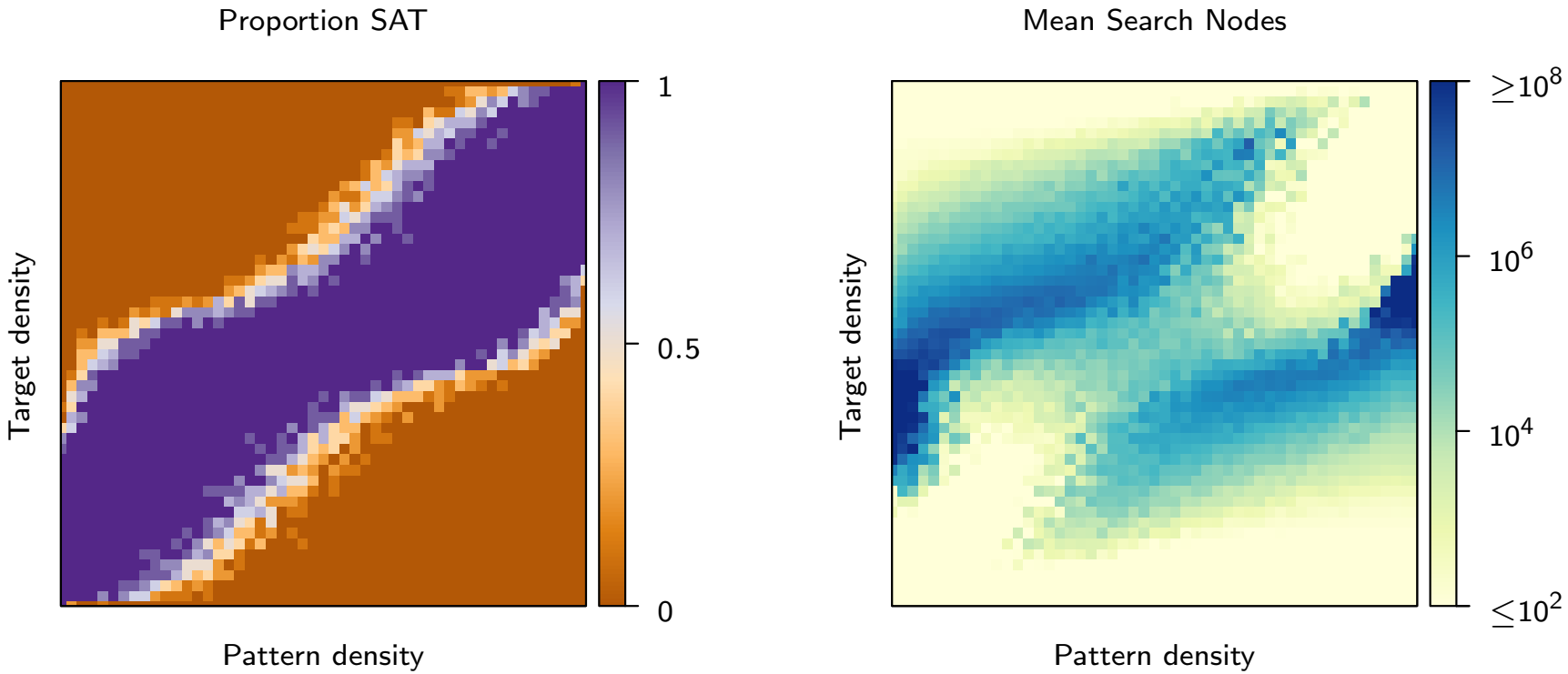
What About Induced?

$G(10, x) \hookrightarrow G(150, y)$



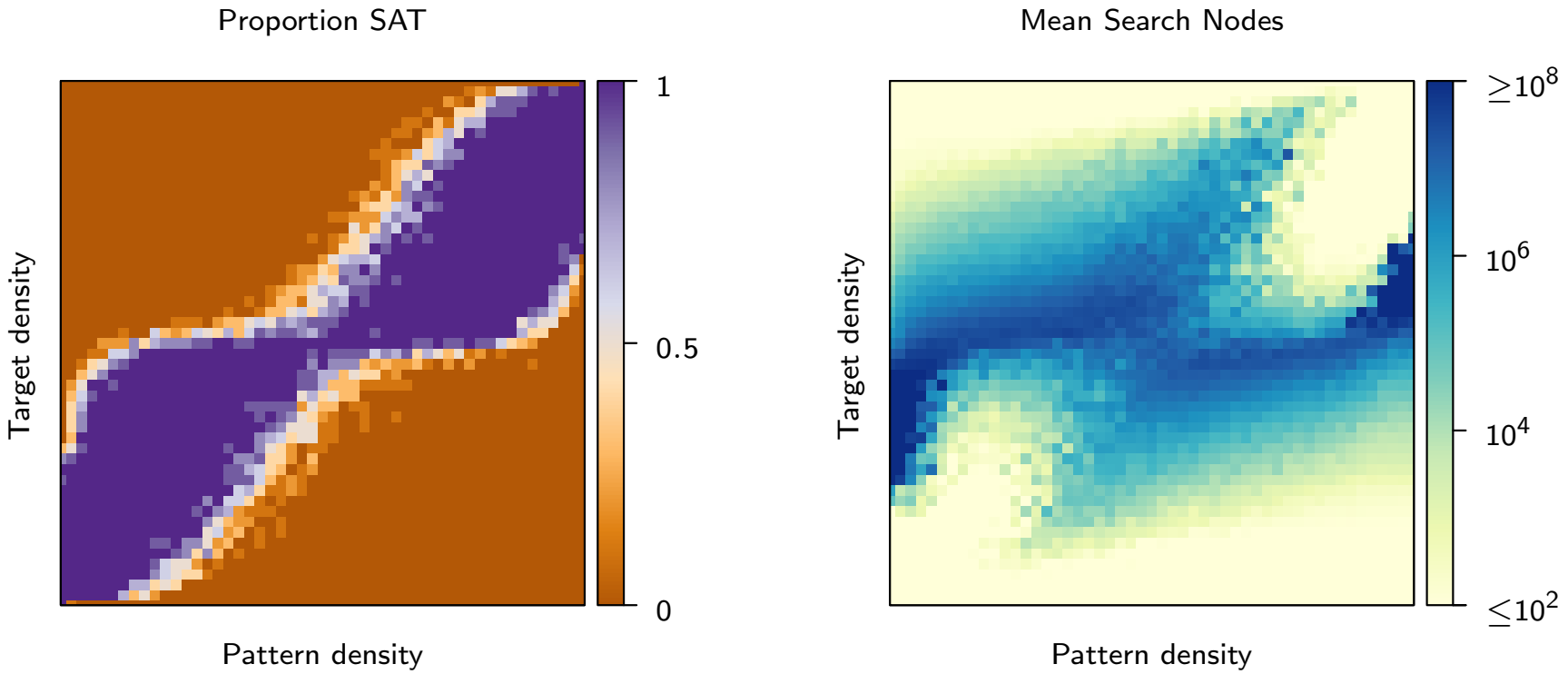
What About Induced?

$G(14, x) \hookrightarrow G(150, y)$



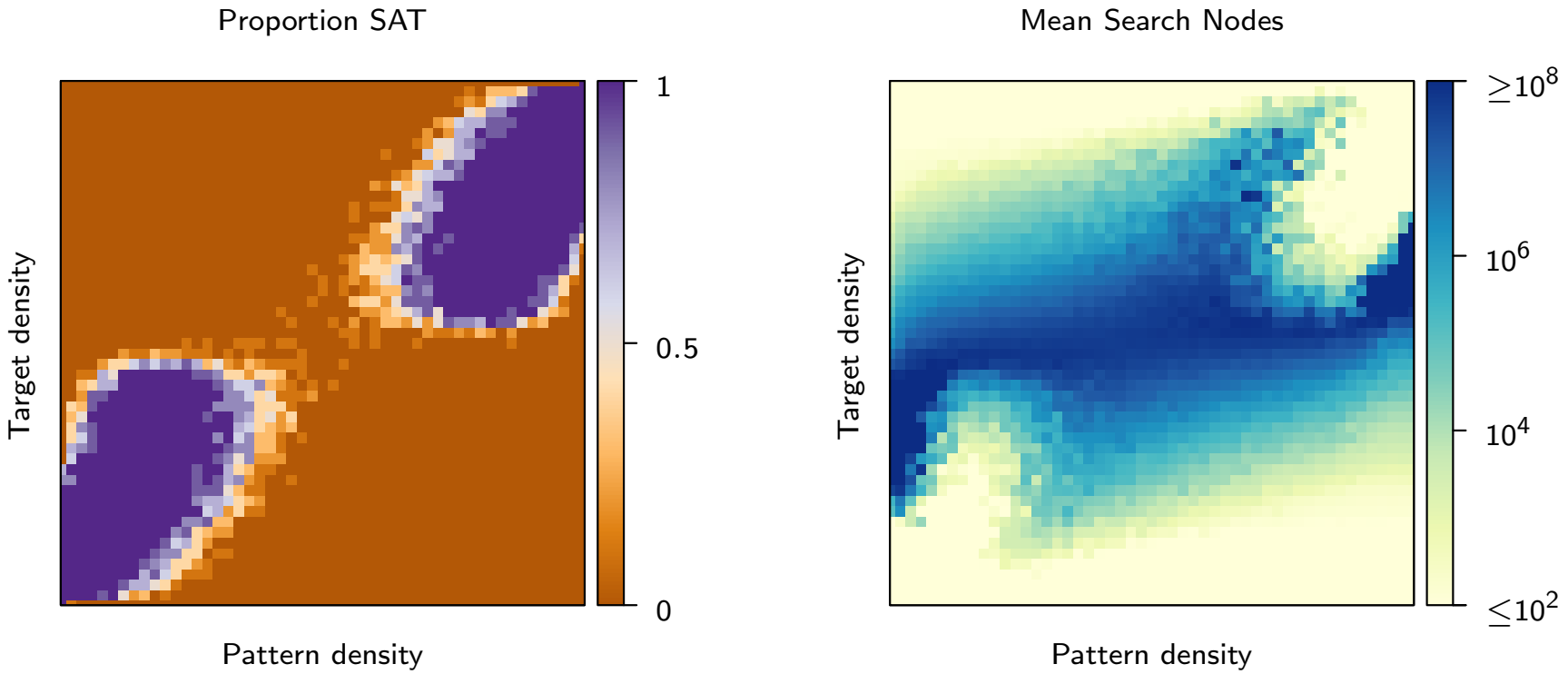
What About Induced?

$G(15, x) \hookrightarrow G(150, y)$



What About Induced?

$G(16, x) \hookrightarrow G(150, y)$



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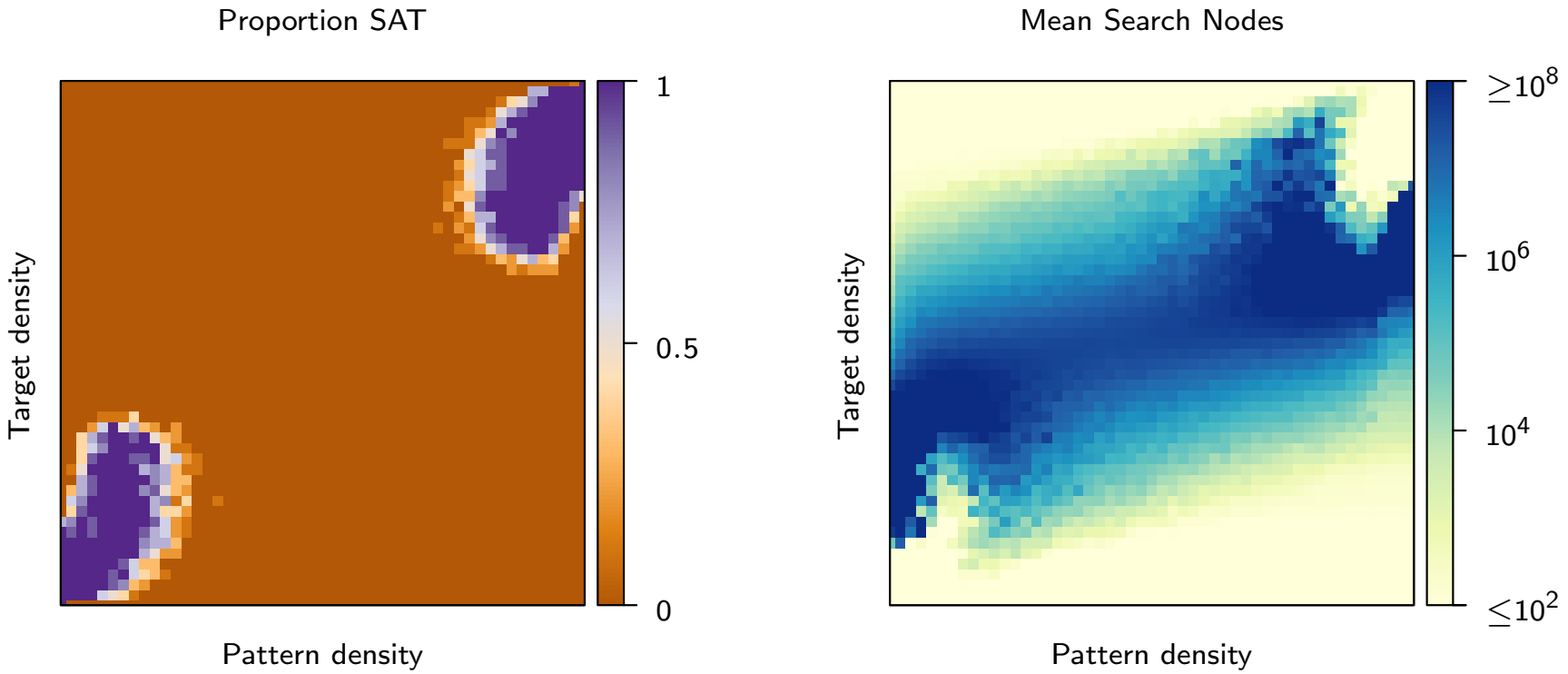
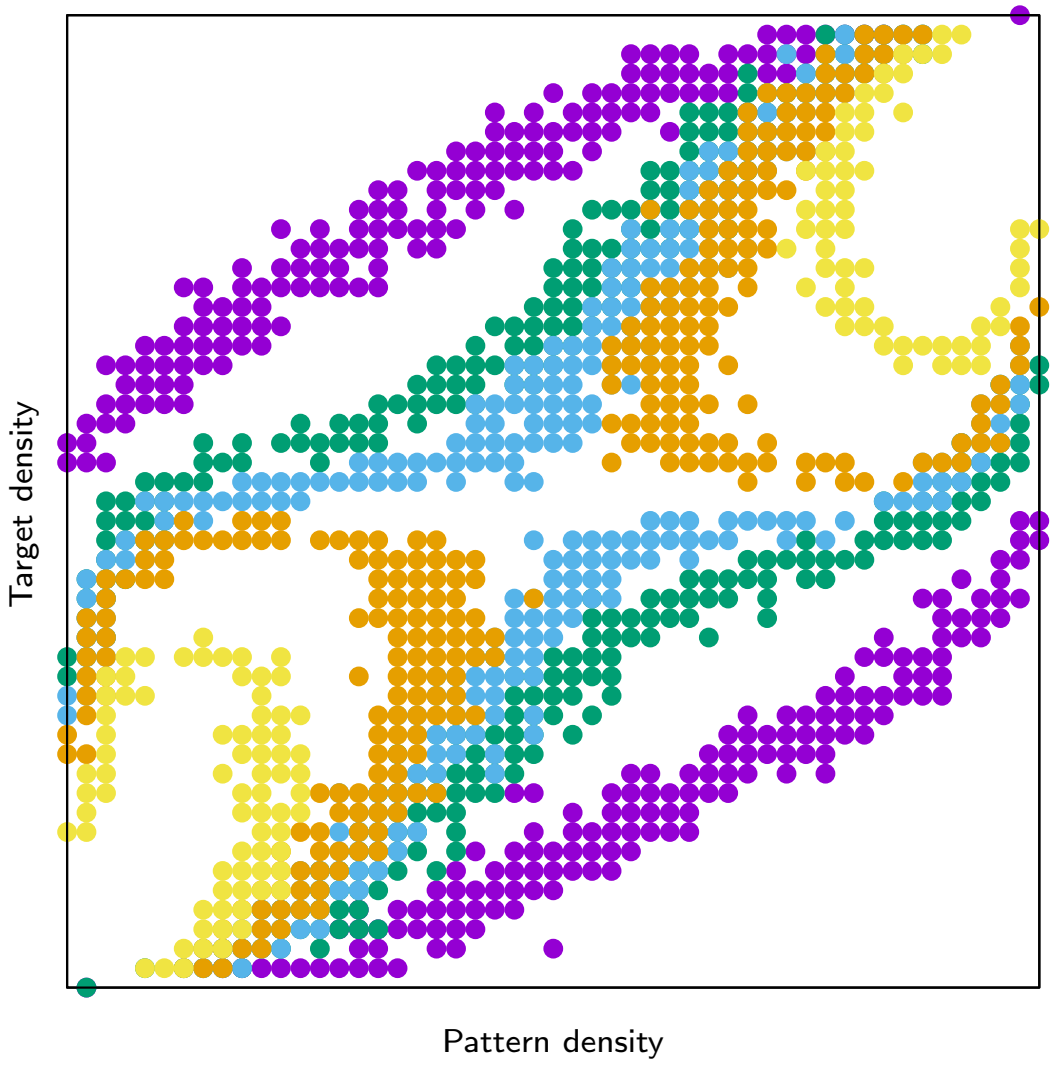


Induced in 3D?



$G(10, x) \hookrightarrow G(150, y)$
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$G(16, x) \hookrightarrow G(150, y)$
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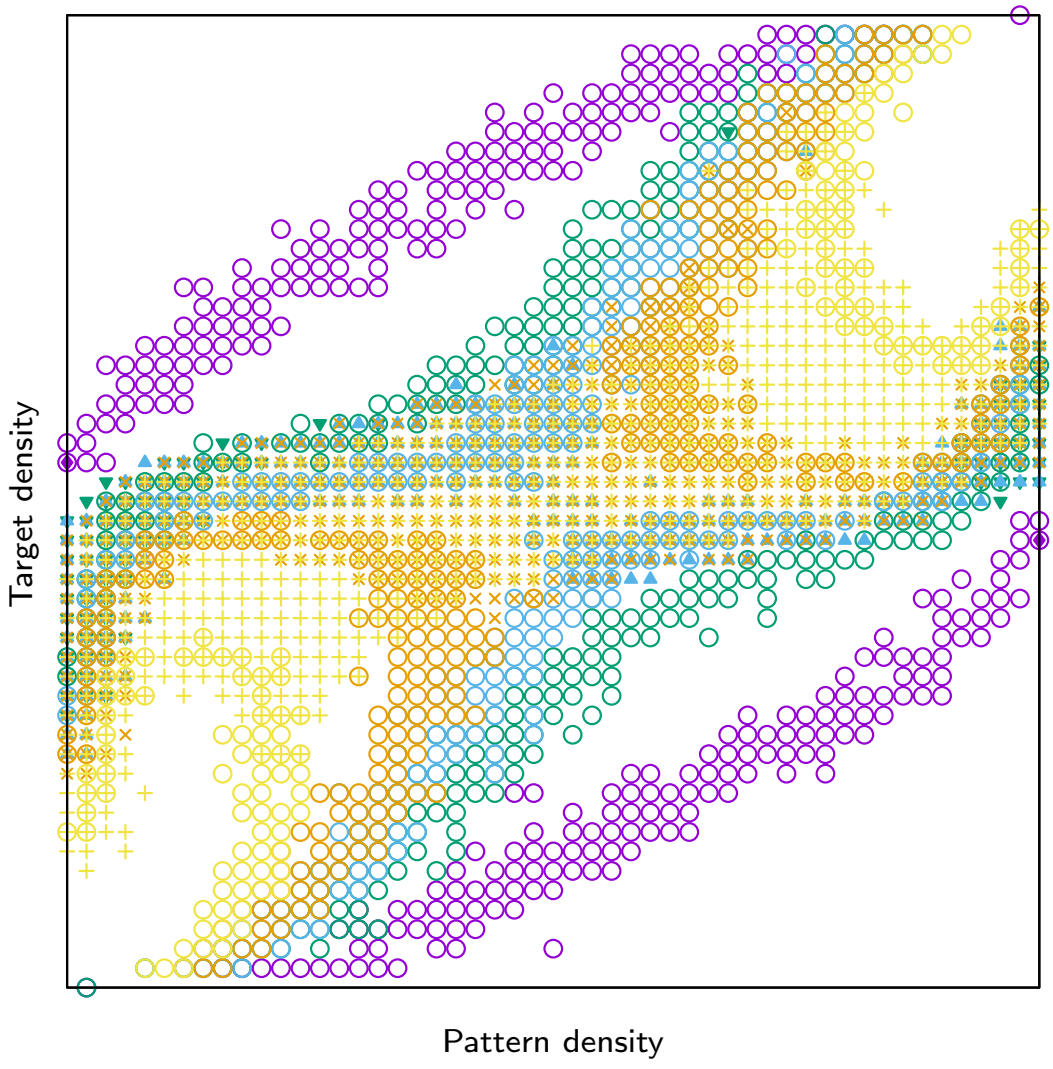

$G(10, x) \hookrightarrow G(150, y)$ (hard)

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$G(20, x) \hookrightarrow G(150, y)$ (hard)

$G(30, x) \hookrightarrow G(150, y)$ (hard)

Other Solvers?

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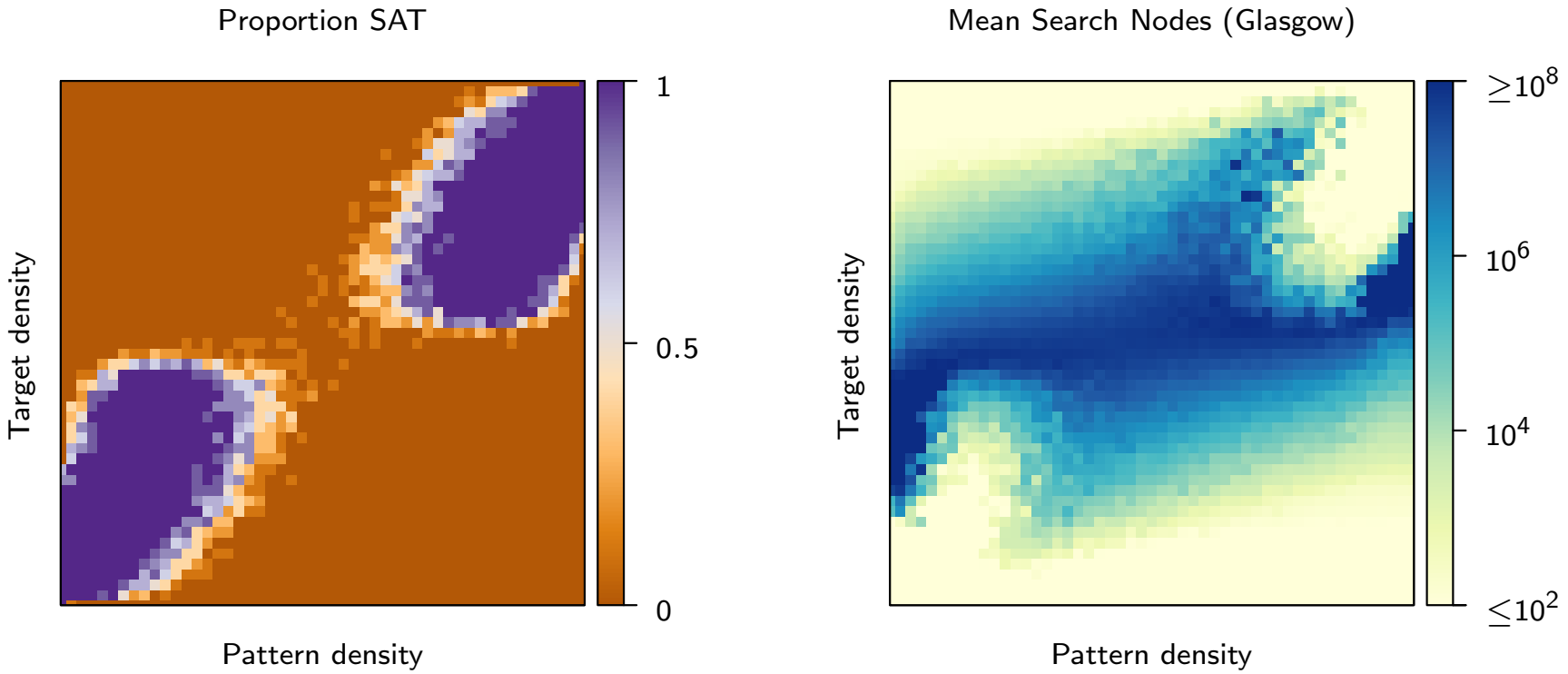

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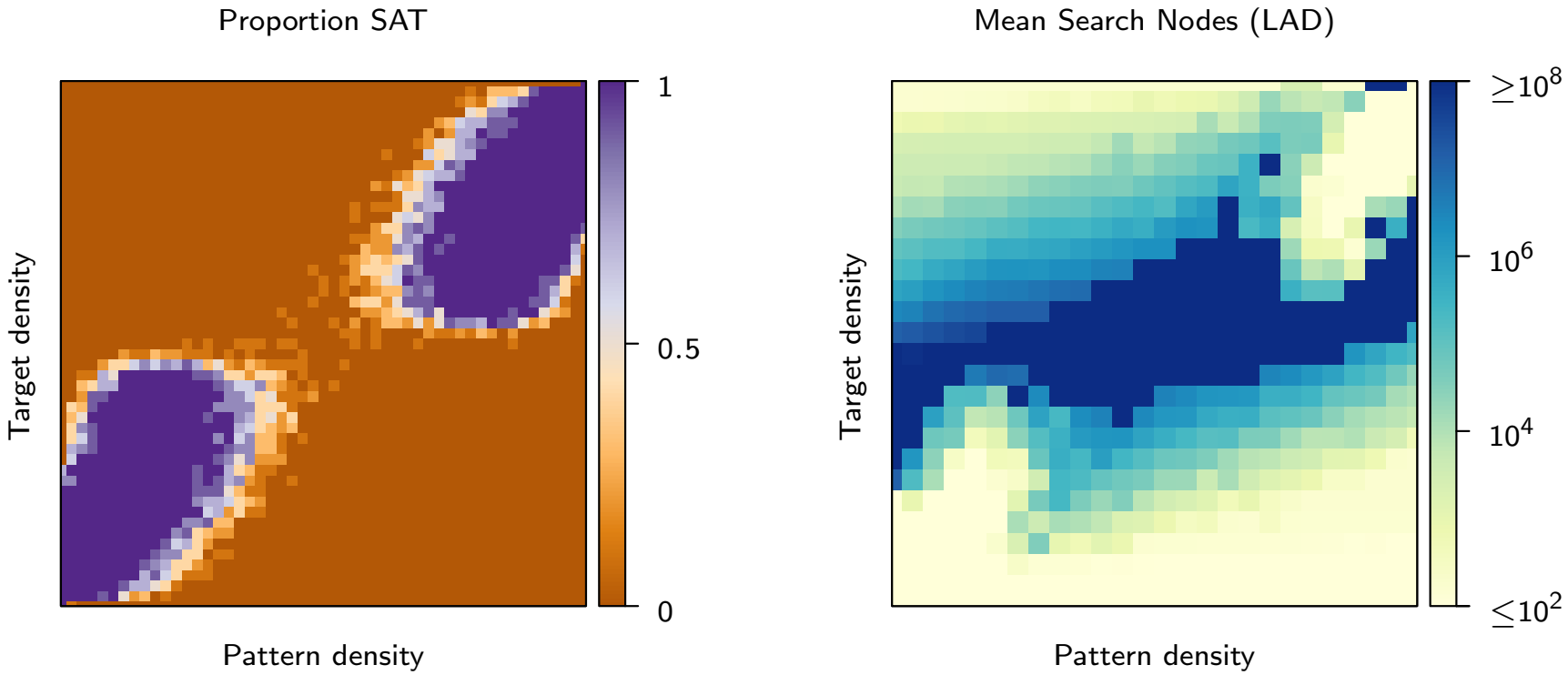

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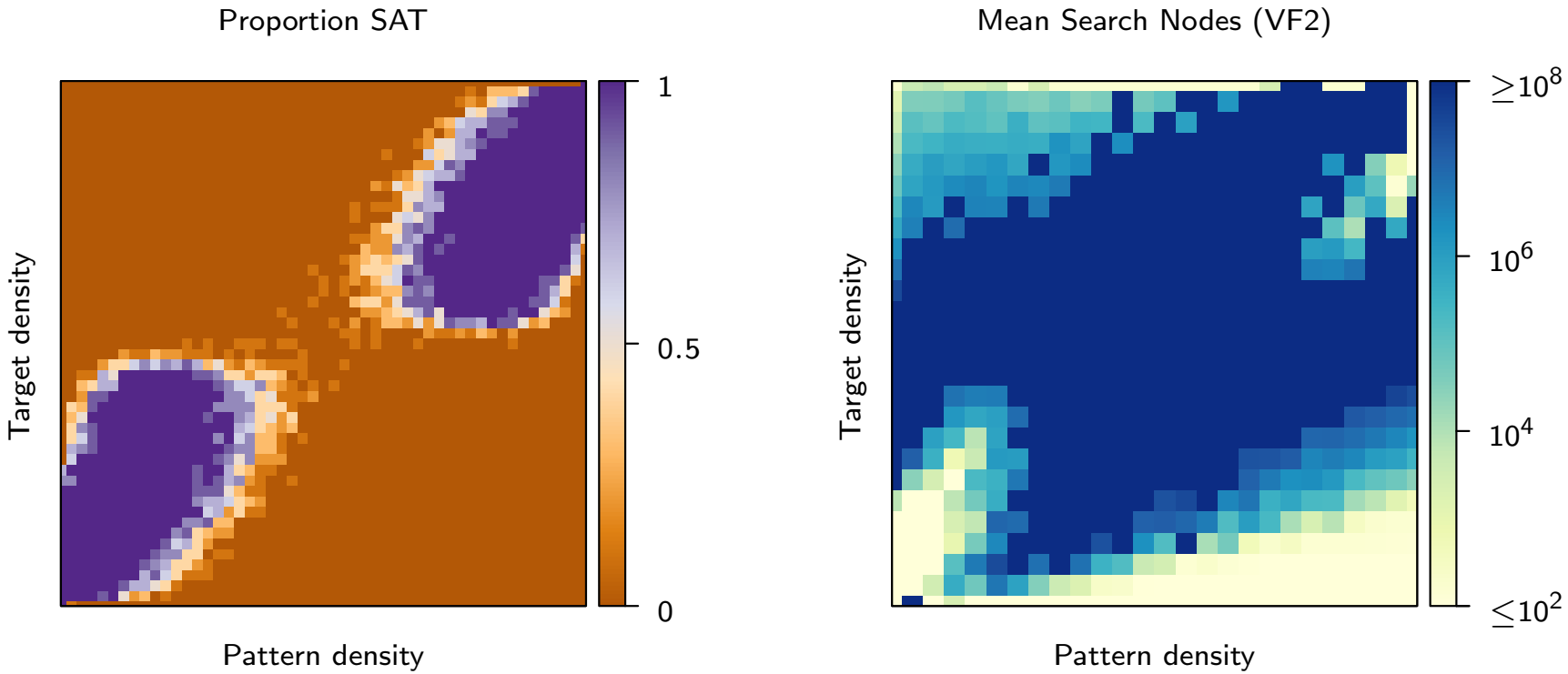
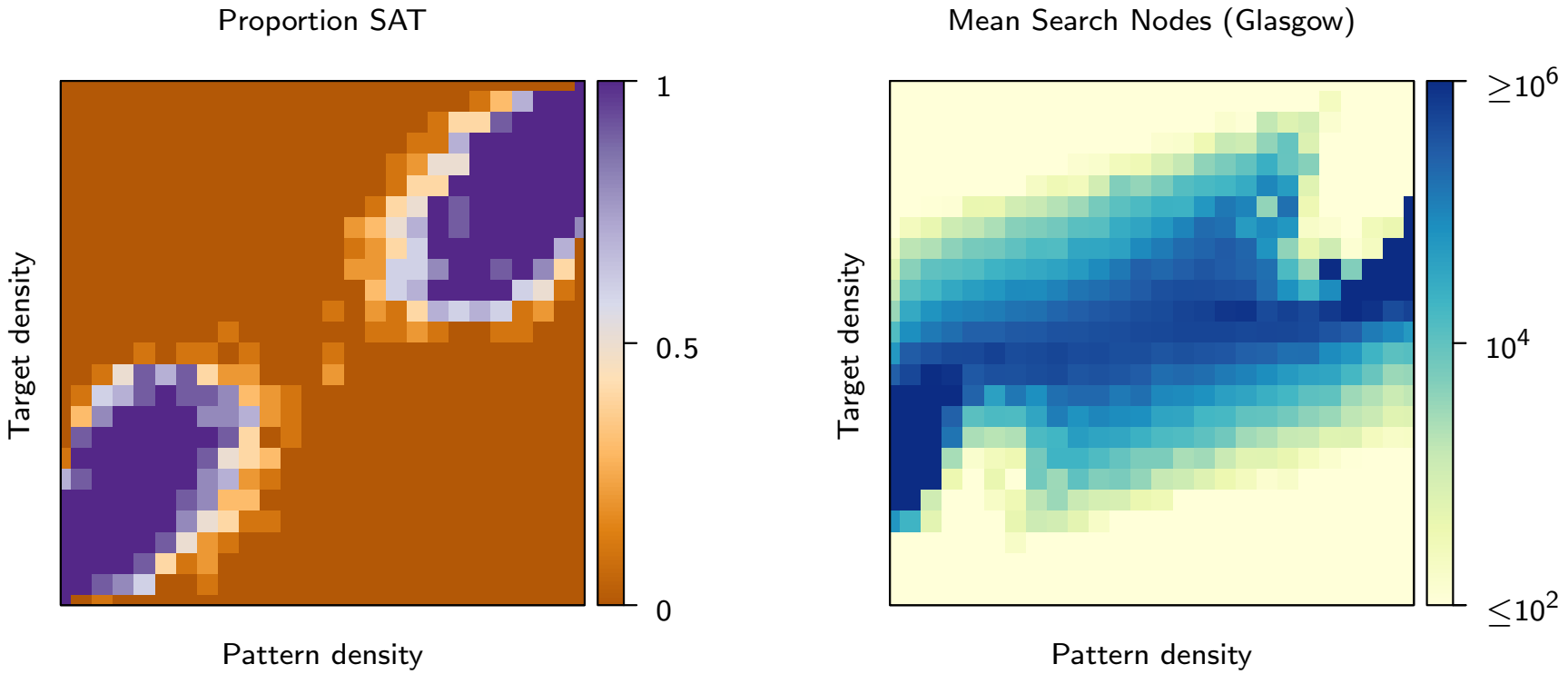


Reductions

$G(14, x) \hookrightarrow G(75, y)$



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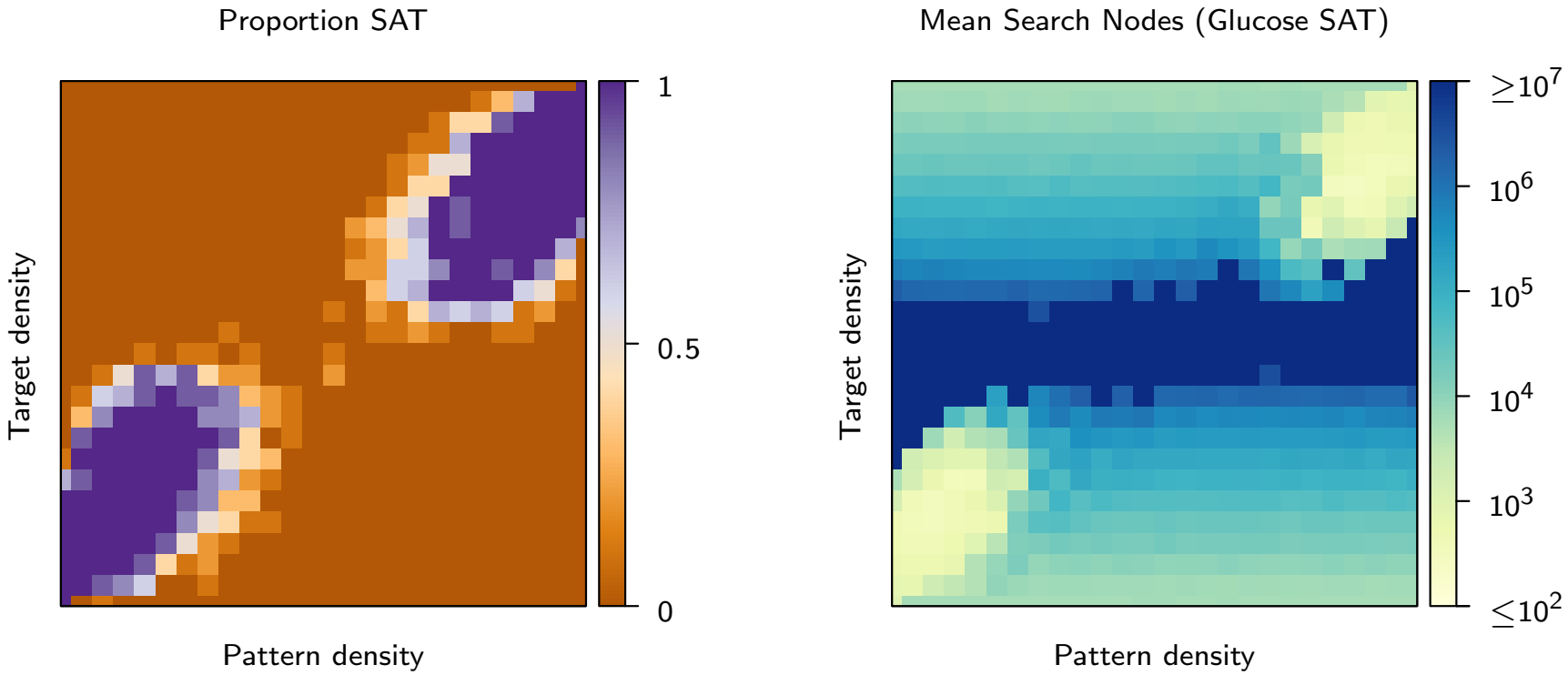

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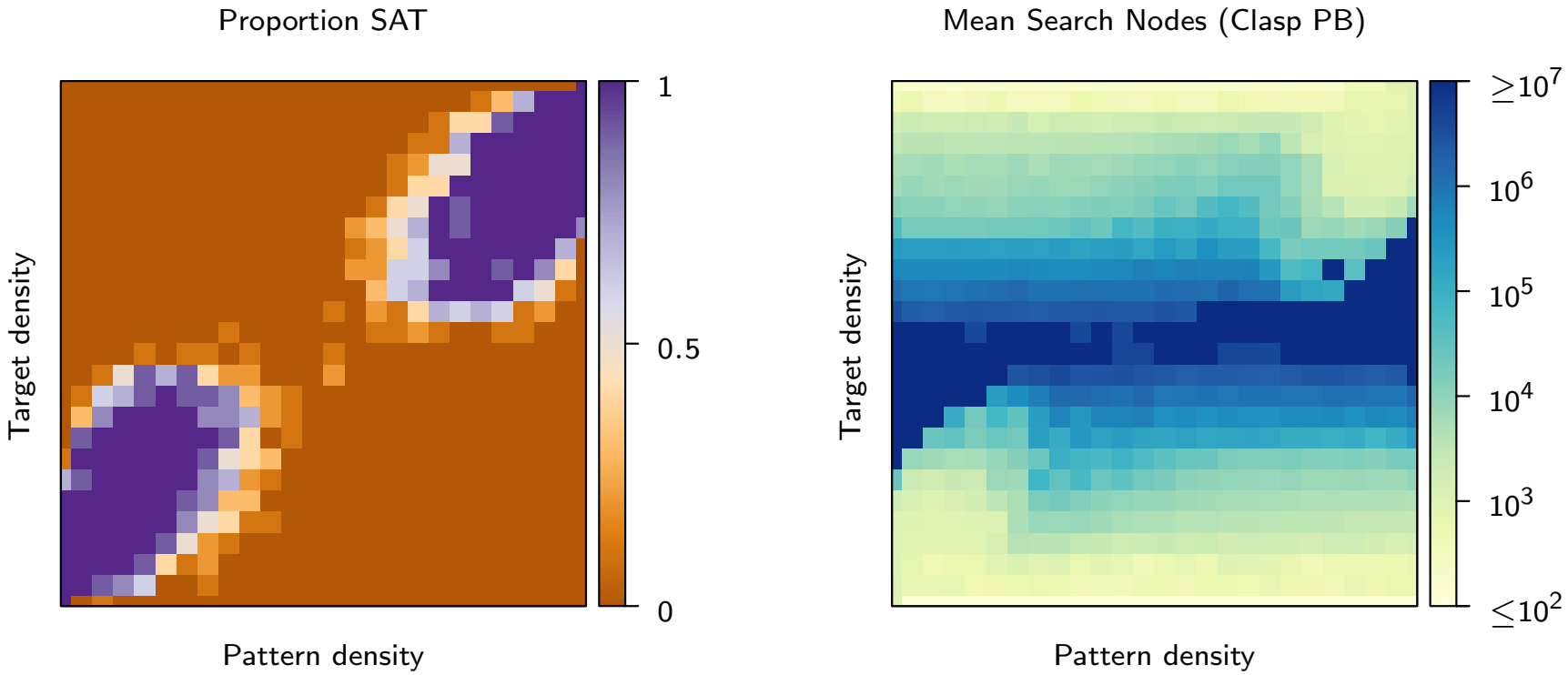

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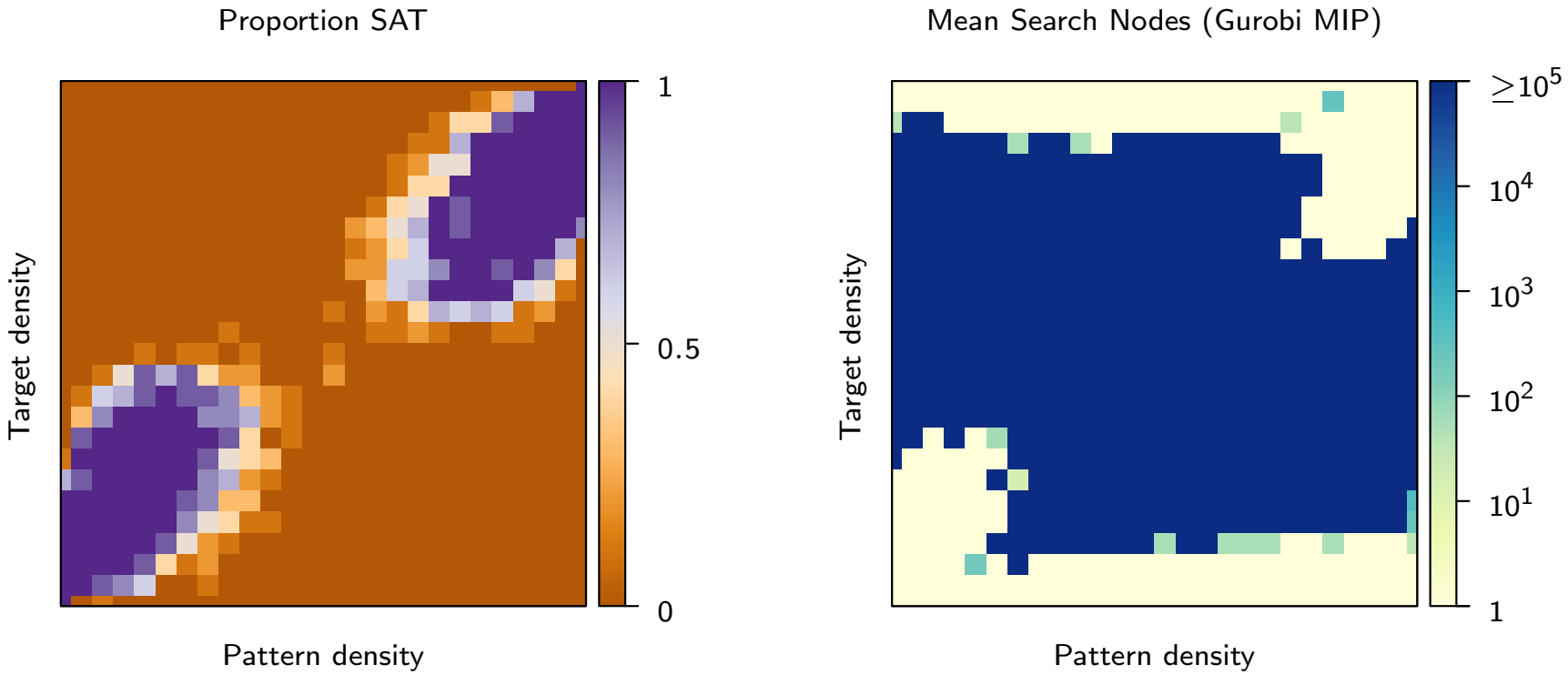

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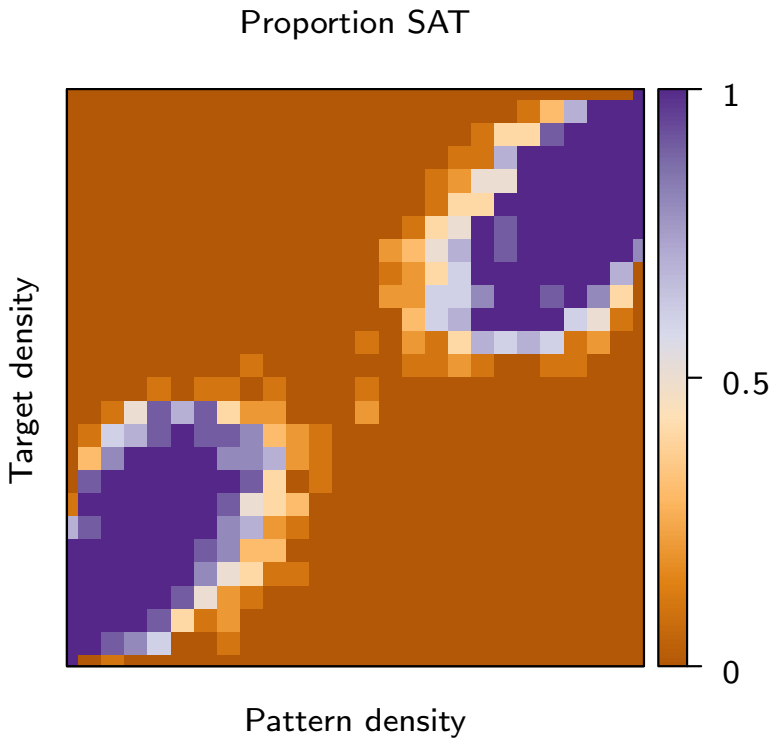

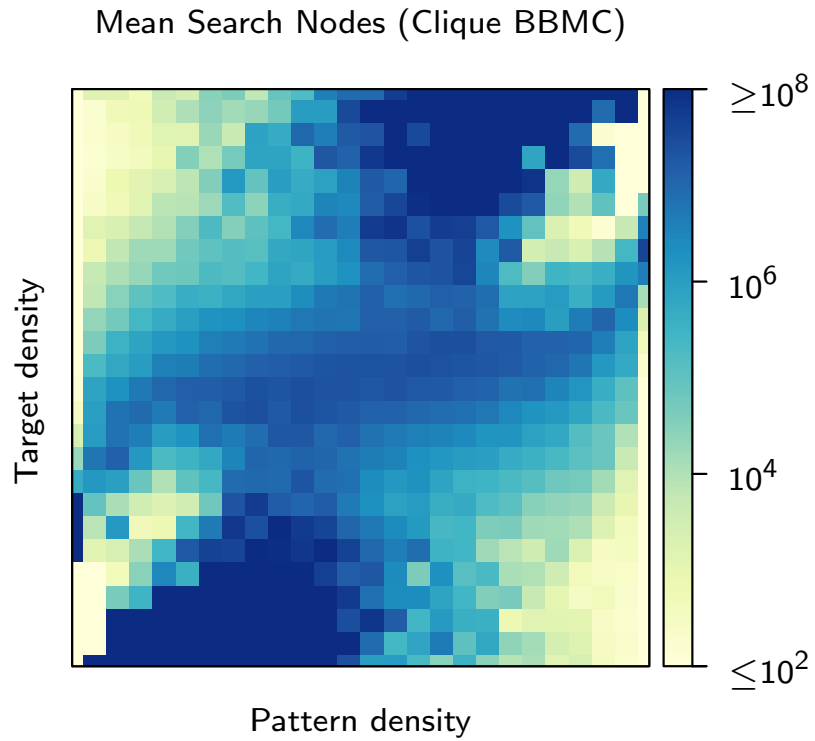

Why the Weird Shape?

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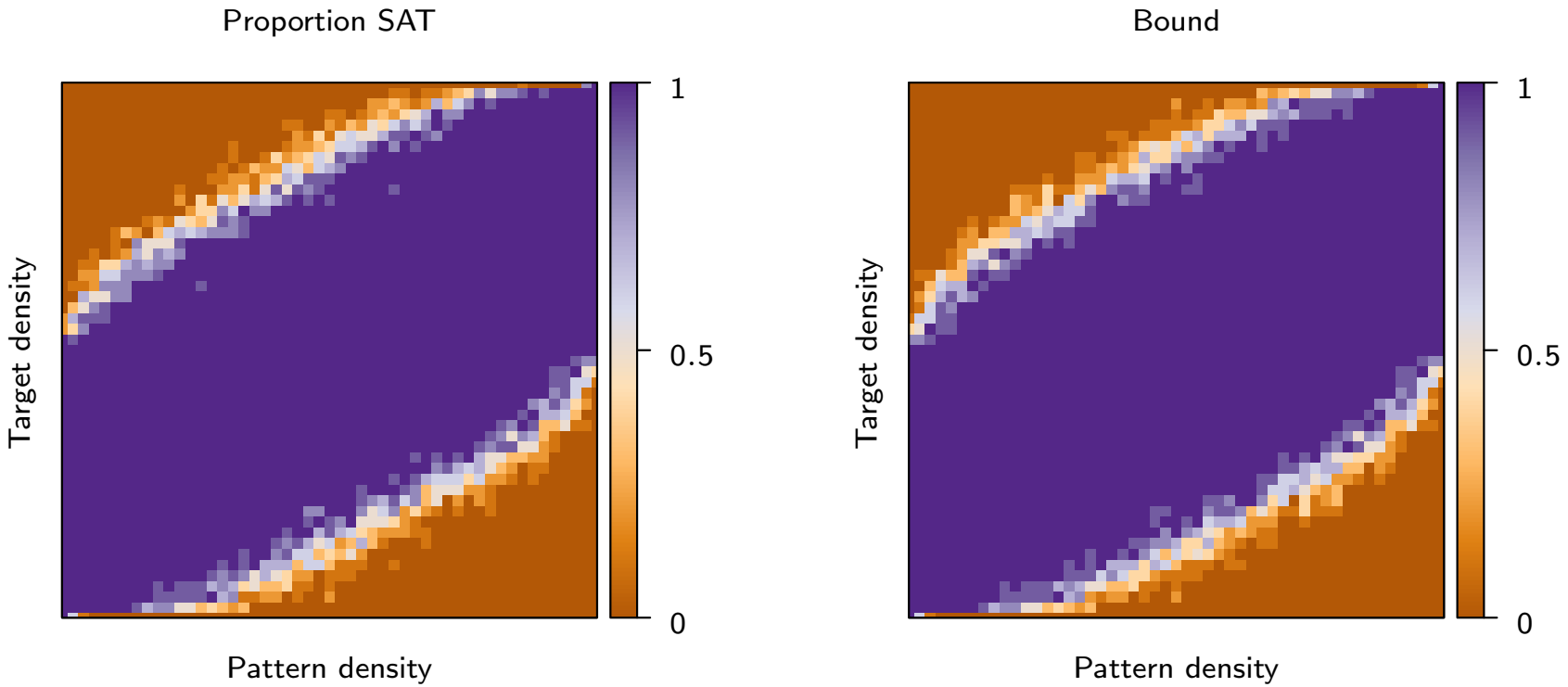

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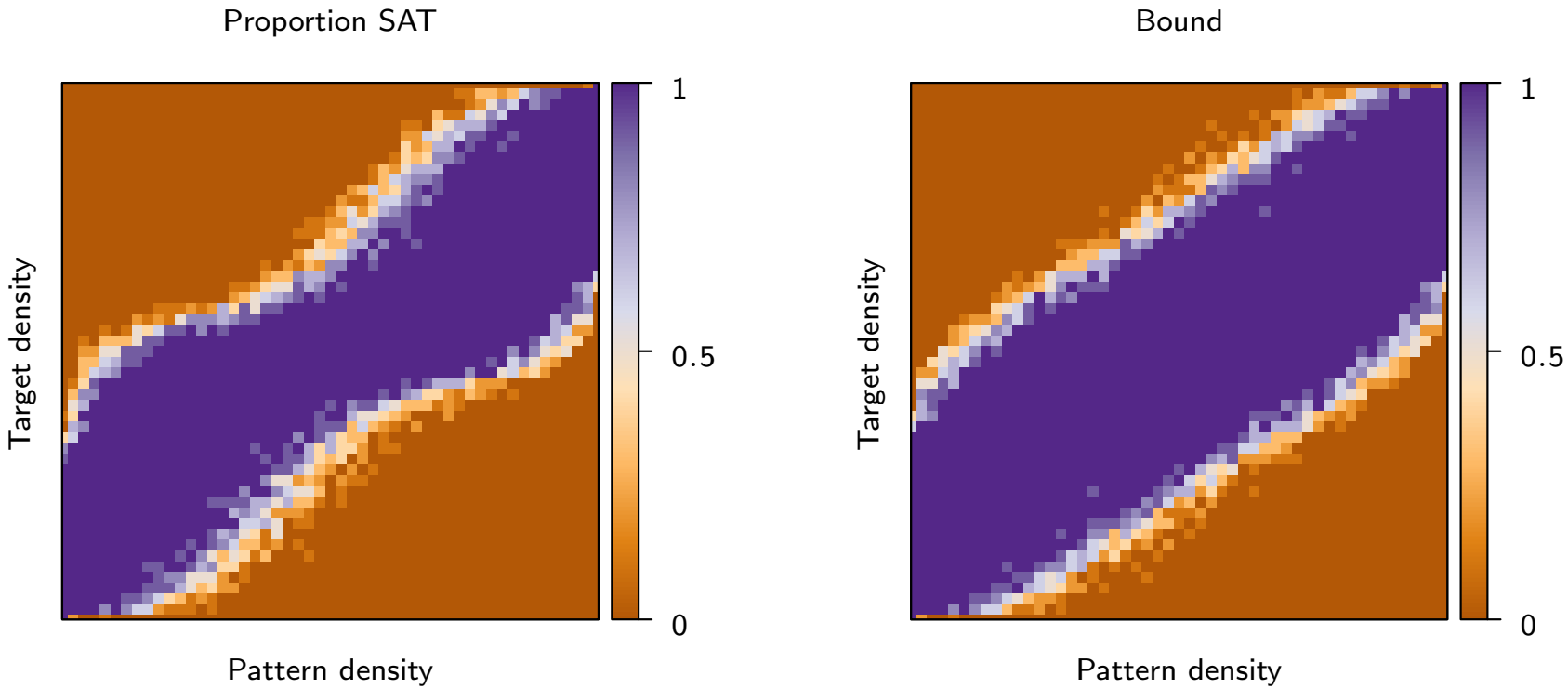

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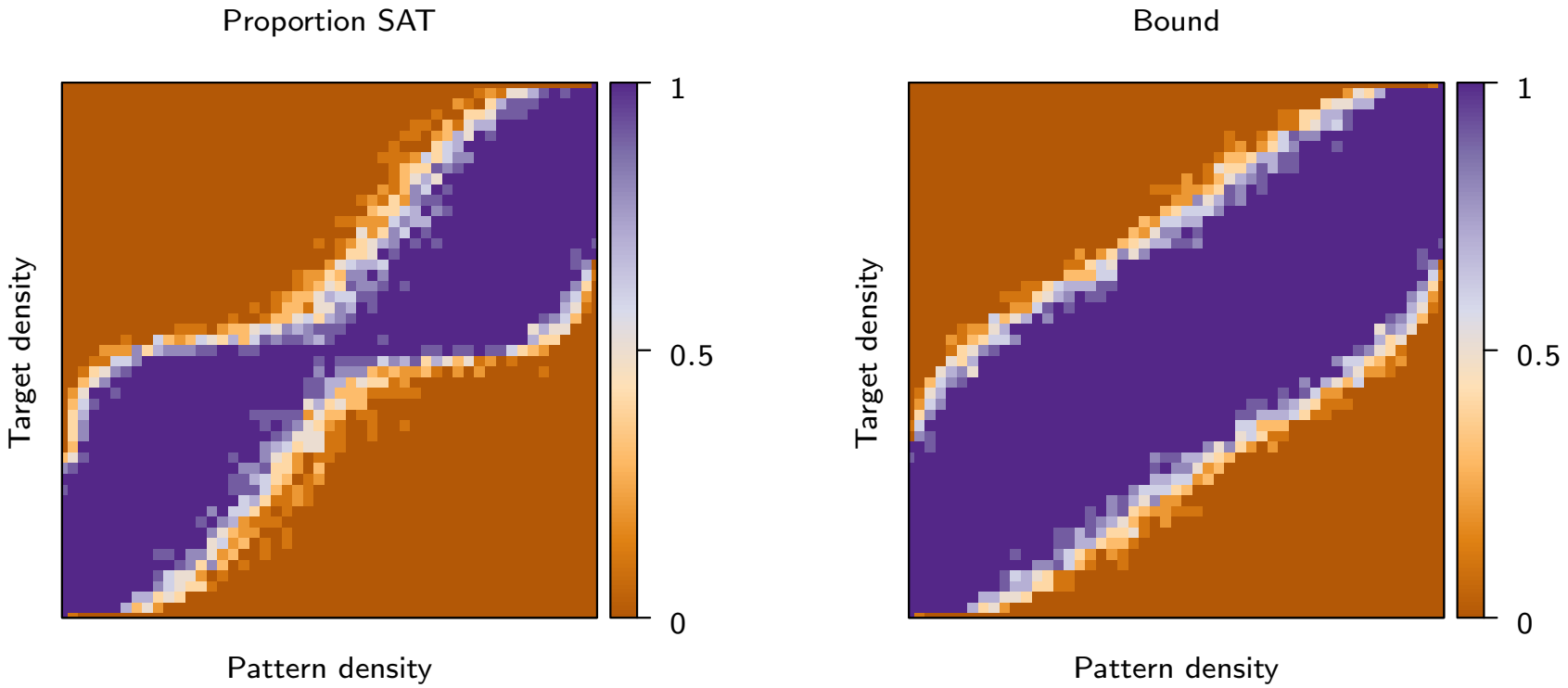

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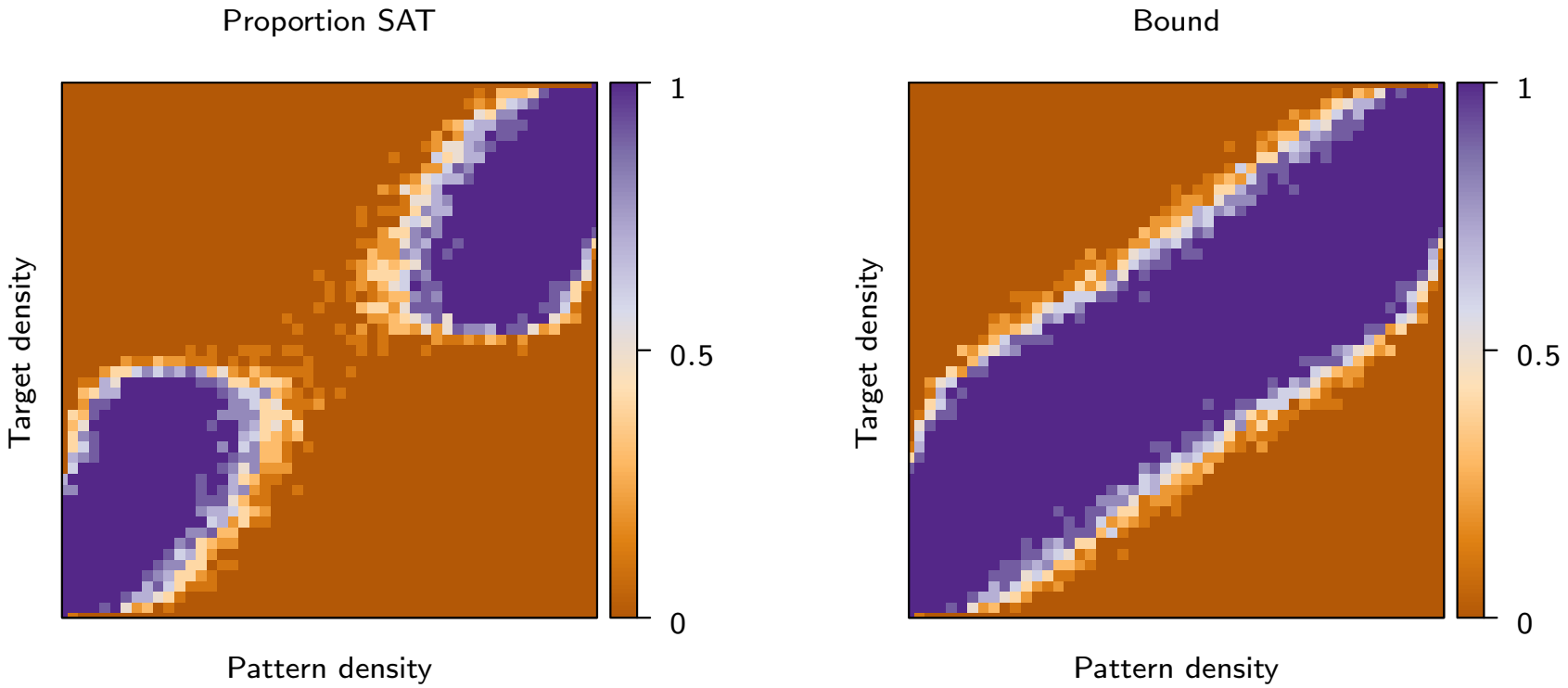

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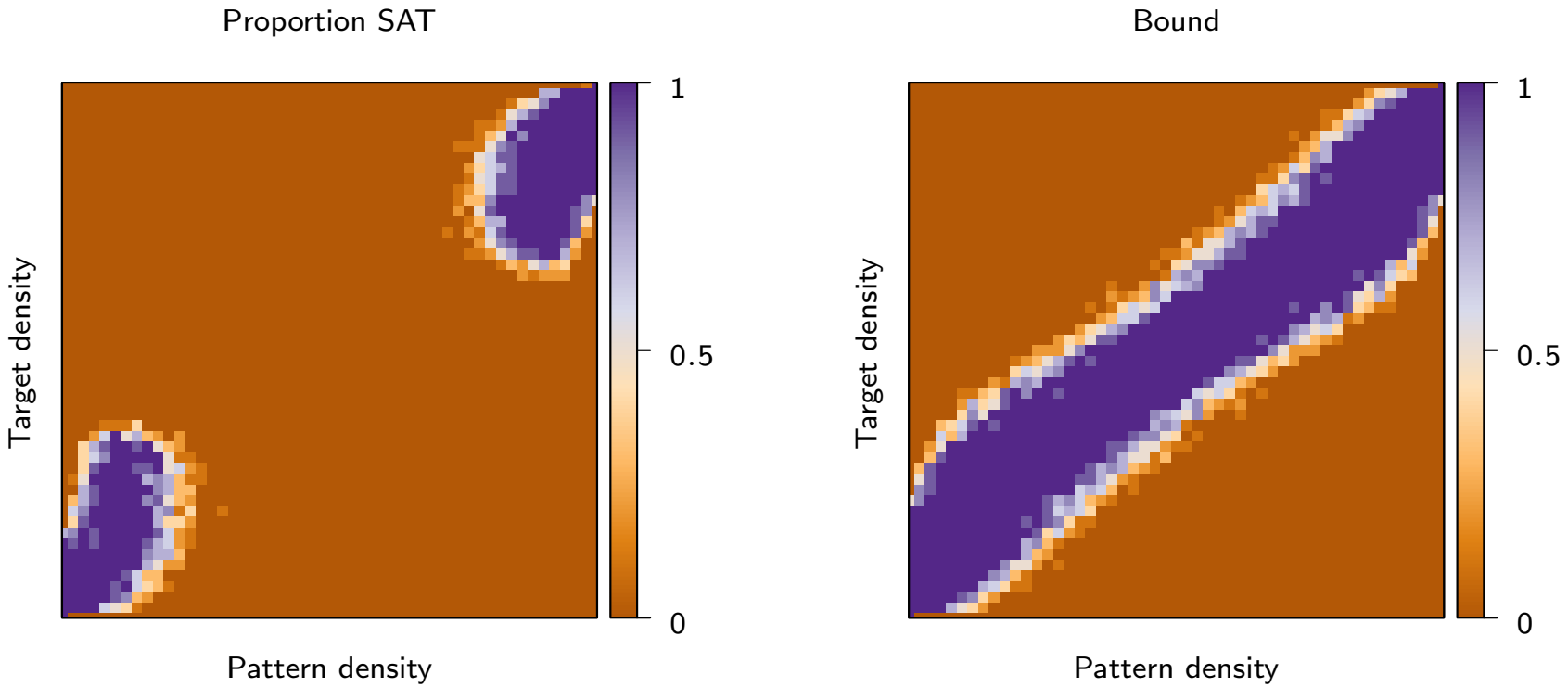

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