Sequential and Parallel Solution-Biased Search for Subgraph Algorithms

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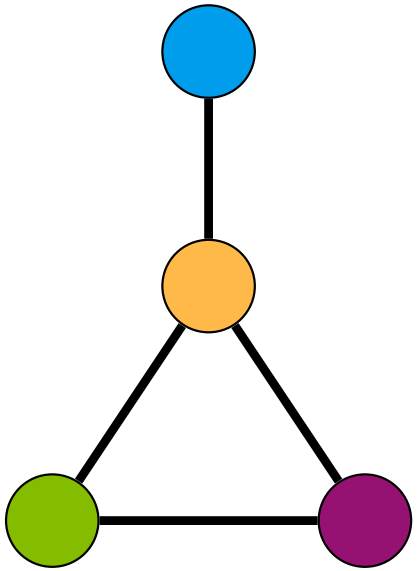
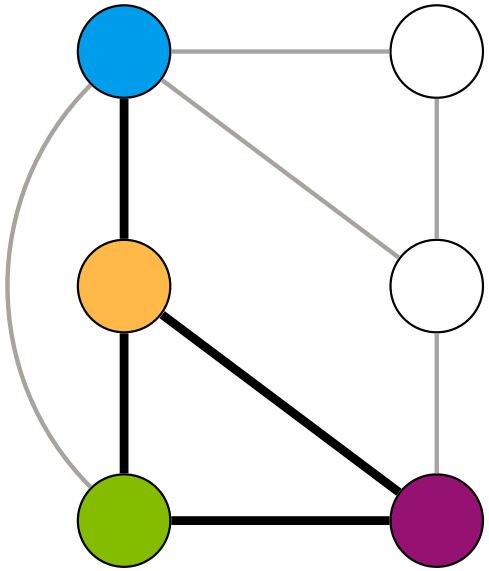
Subgraph Isomorphism





Subgraph Finding, as a Constraint Program

- A variable for each pattern vertex. The domains are all of the target vertices.
- At least two sets of constraints:
  - Adjacent pairs of vertices must be mapped to adjacent pairs of vertices.
  - All different.
- Then we get clever:
  - Extra constraints about degrees, paths, …
  - Very good variable- and value-ordering heuristics

The Glasgow Subgraph Solver

`https://github.com/ciaranm/glasgow-subgraph-solver`

- A CP style solver specifically for subgraph algorithms.
- Subgraph isomorphism, and all its variants (induced / non-induced, homomorphism, locally injective, labels, side constraints, directed, ...).
- Also special algorithms for clique.

Benchmark Instances

- 14,621 instances from Christine Solnon's collection:
  - Randomly generated with different models.
  - Real-world graphs.
  - Computer vision problems.
  - Biochemistry problems.
  - Phase transition instances.
- At least…
  - $\geq$ 2,110 satisfiable.
  - $\geq$ 12,322 unsatisfiable.
- A lot of them are very easy for good algorithms.

Hardware

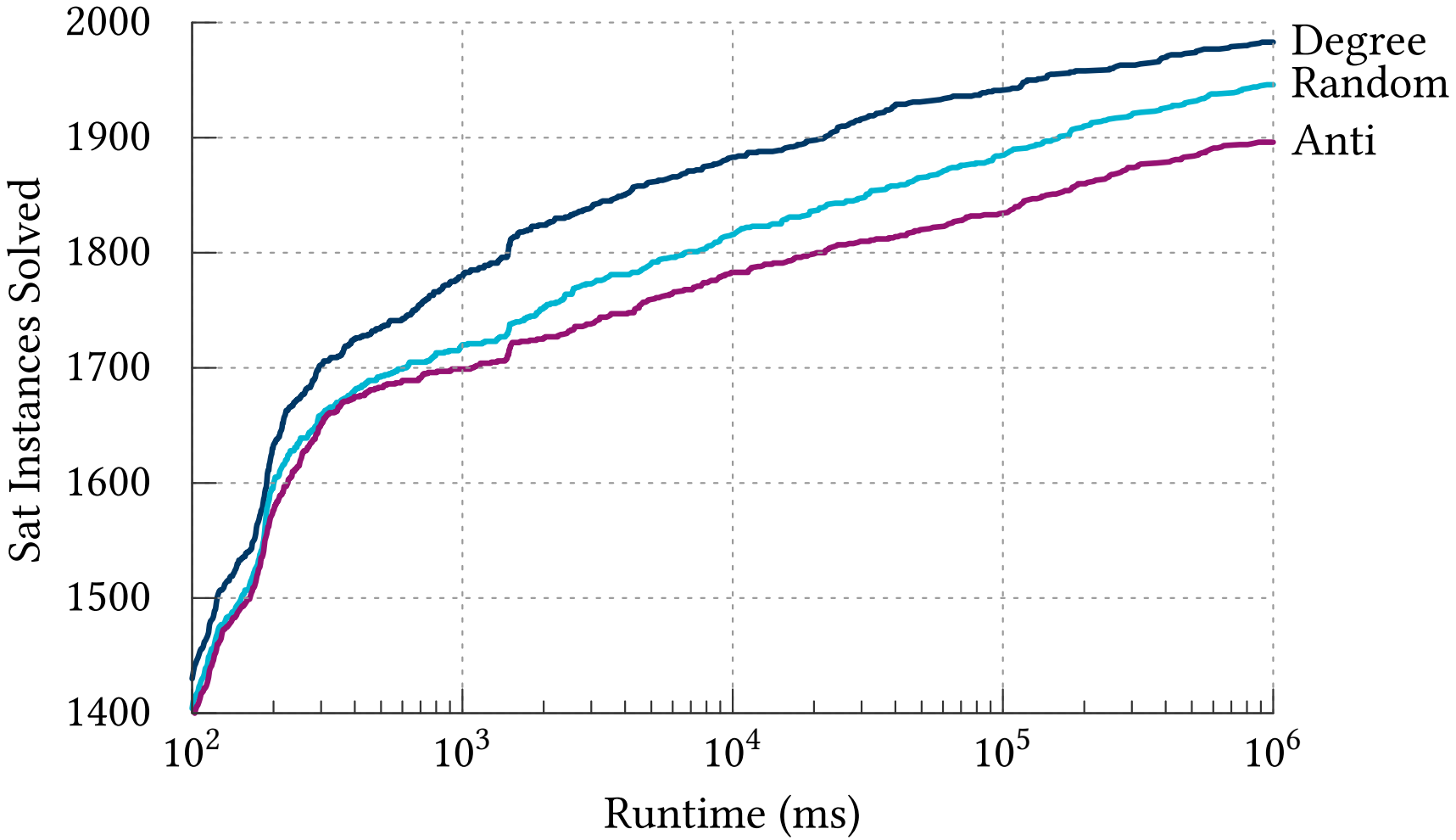
- HPC, optimised for throughput not reproducibility.[1]
- Dual Intel Xeon E5-2695 v4 CPUs, 2×18 cores
- 256GBytes RAM
- GCC 7.2.0
- C++ native threads, SGI MPT MPI

[1]This work used the Cirrus UK National Tier-2 HPC Service at EPCC (http://www.cirrus.ac.uk) funded by the University of Edinburgh and EPSRC (EP/P020267/1)

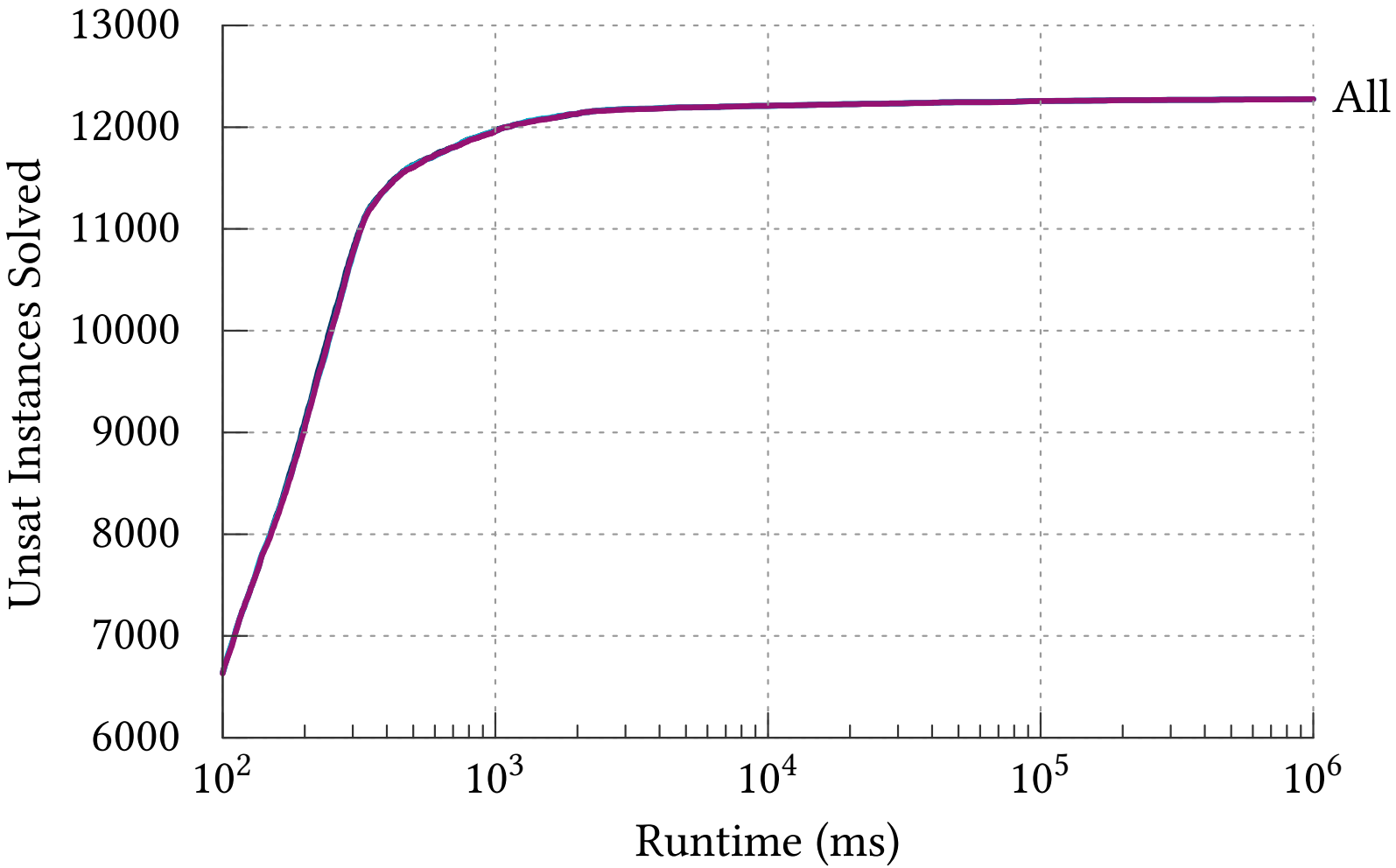
Value-Ordering Heuristics

- Largest target degree first.

Sanity Check



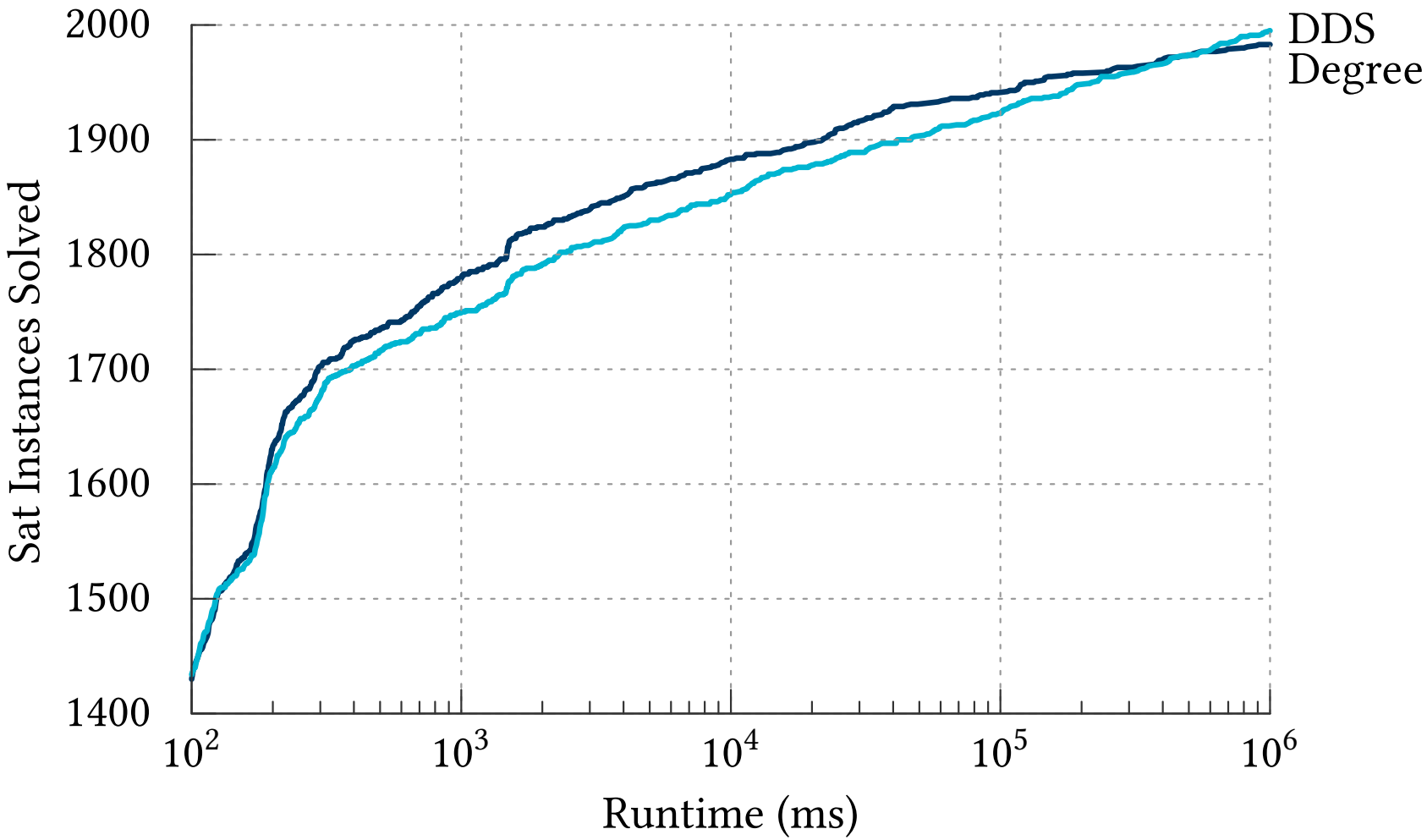
Sanity Check



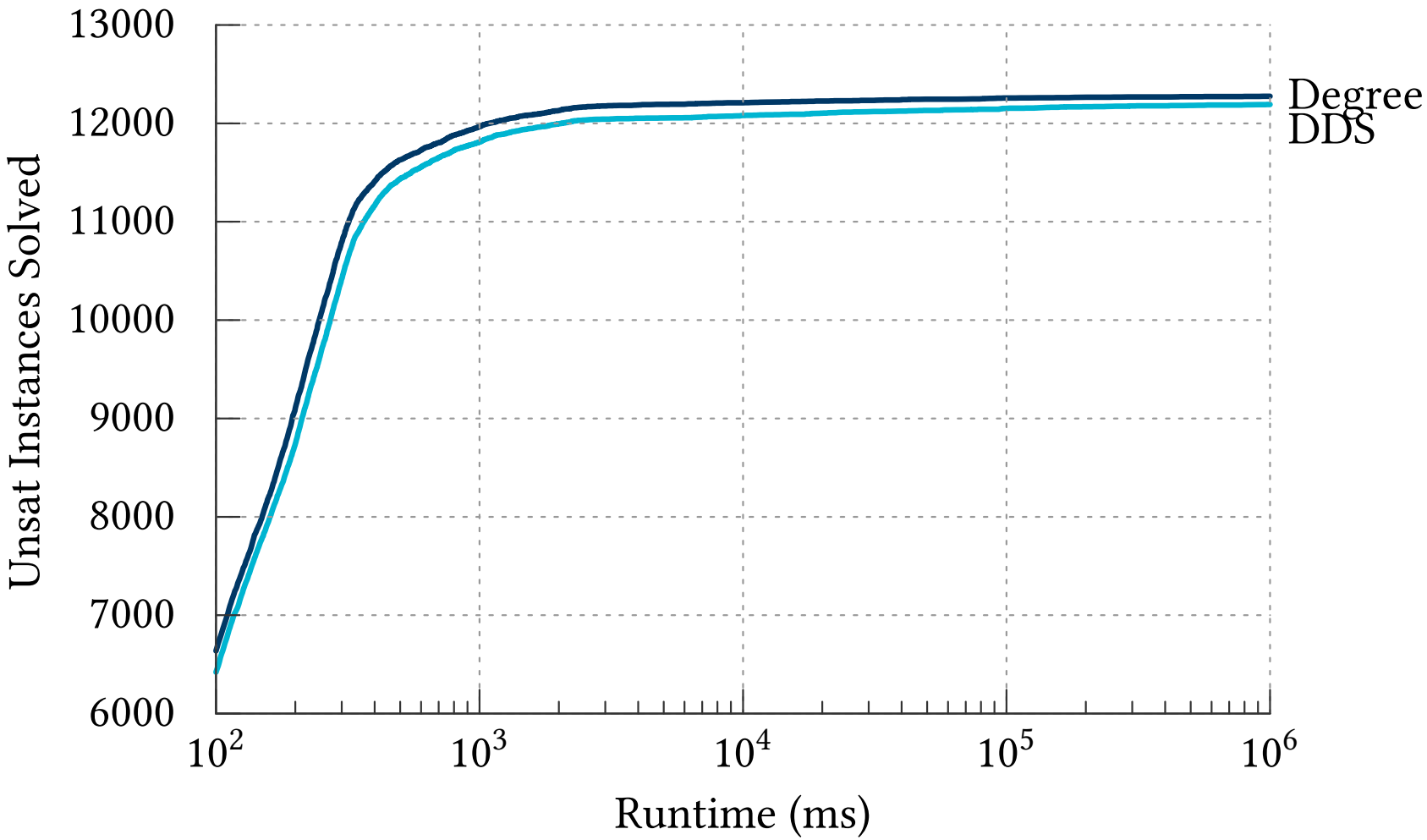
However…

- What if several vertices have the same degree?
- Is a vertex of degree 10 really that much better than a vertex of degree 9?

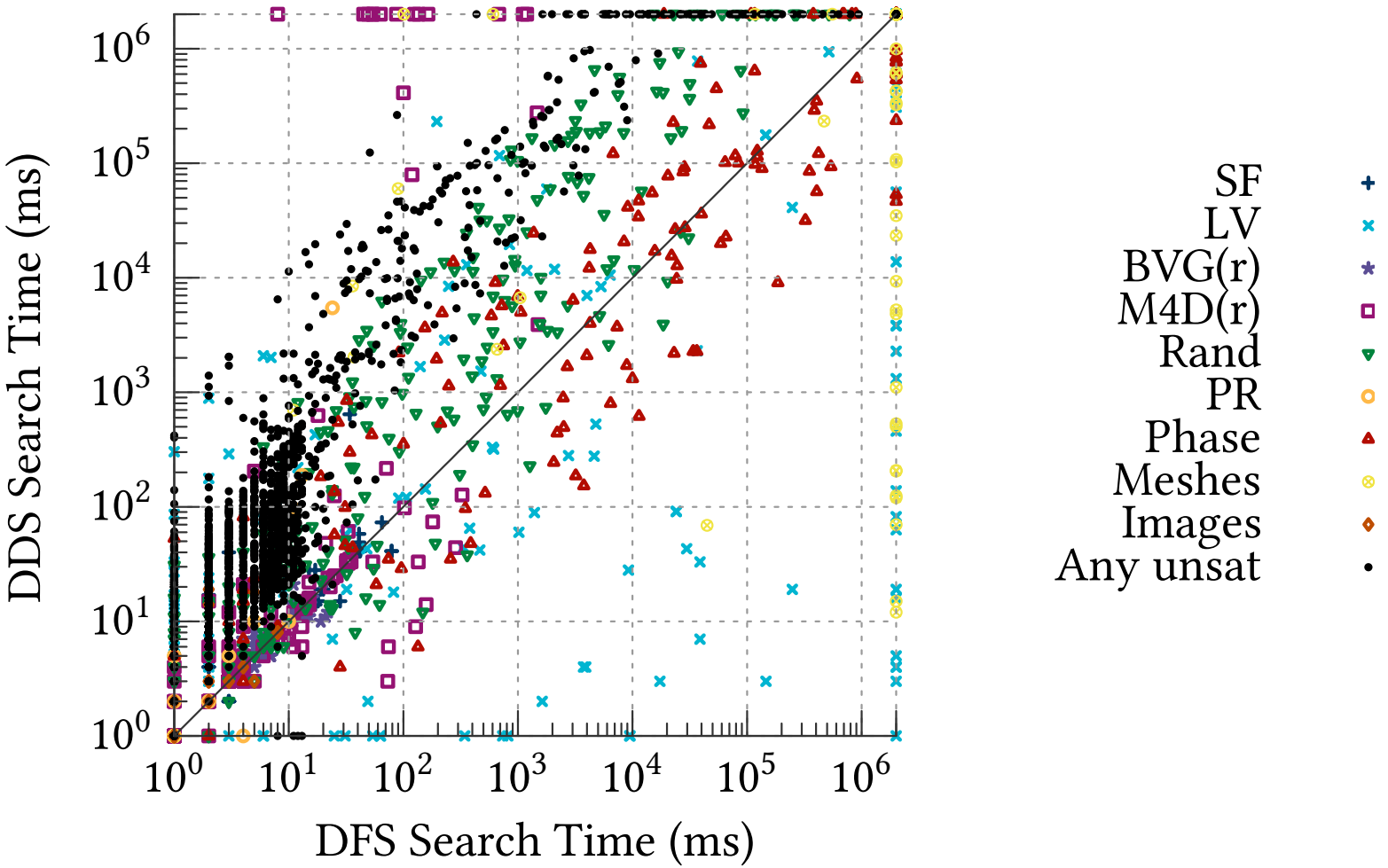
Discrepancy Search?



Discrepancy Search?



Discrepancy Search?



Value-Ordering Heuristics as Distributions

- Traditional view: value-ordering defines a search order.
- New view: value-ordering defines what proportion of the search effort should be spent on different subproblems.

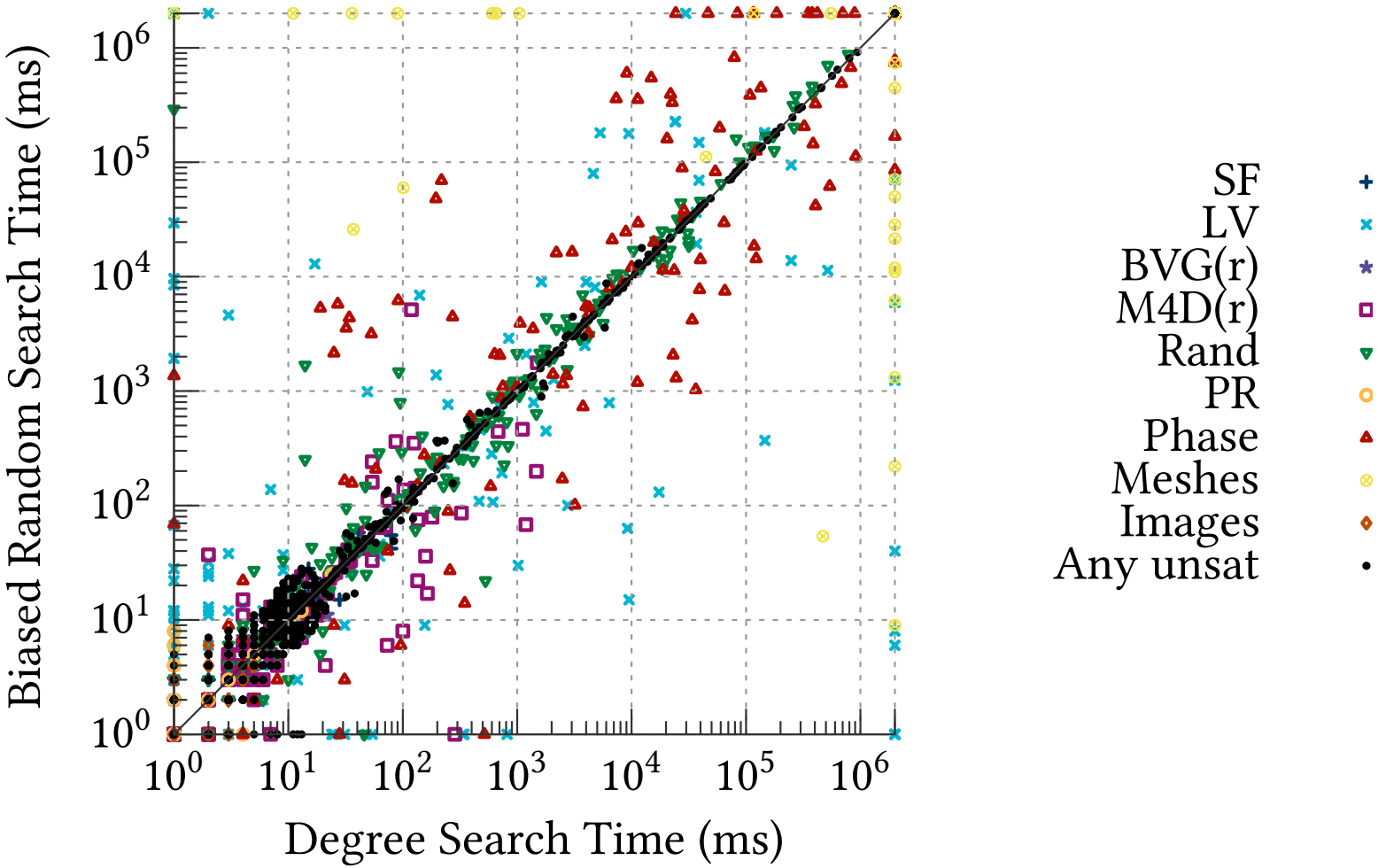
A Slightly Random Value-Ordering Heuristic

- For a fixed domain D_v, pick a vertex v' from a domain D_v with probability

$$p(v') = \frac{2^{\deg(v')}}{\sum_{w \in D_v} 2^{\deg(w)}}$$

- Equally likely to pick between two vertices of degree d.
- Twice as likely to select a vertex of degree d than a vertex of degree $d - 1$.
- In the paper: solution density and expected distribution of solutions.

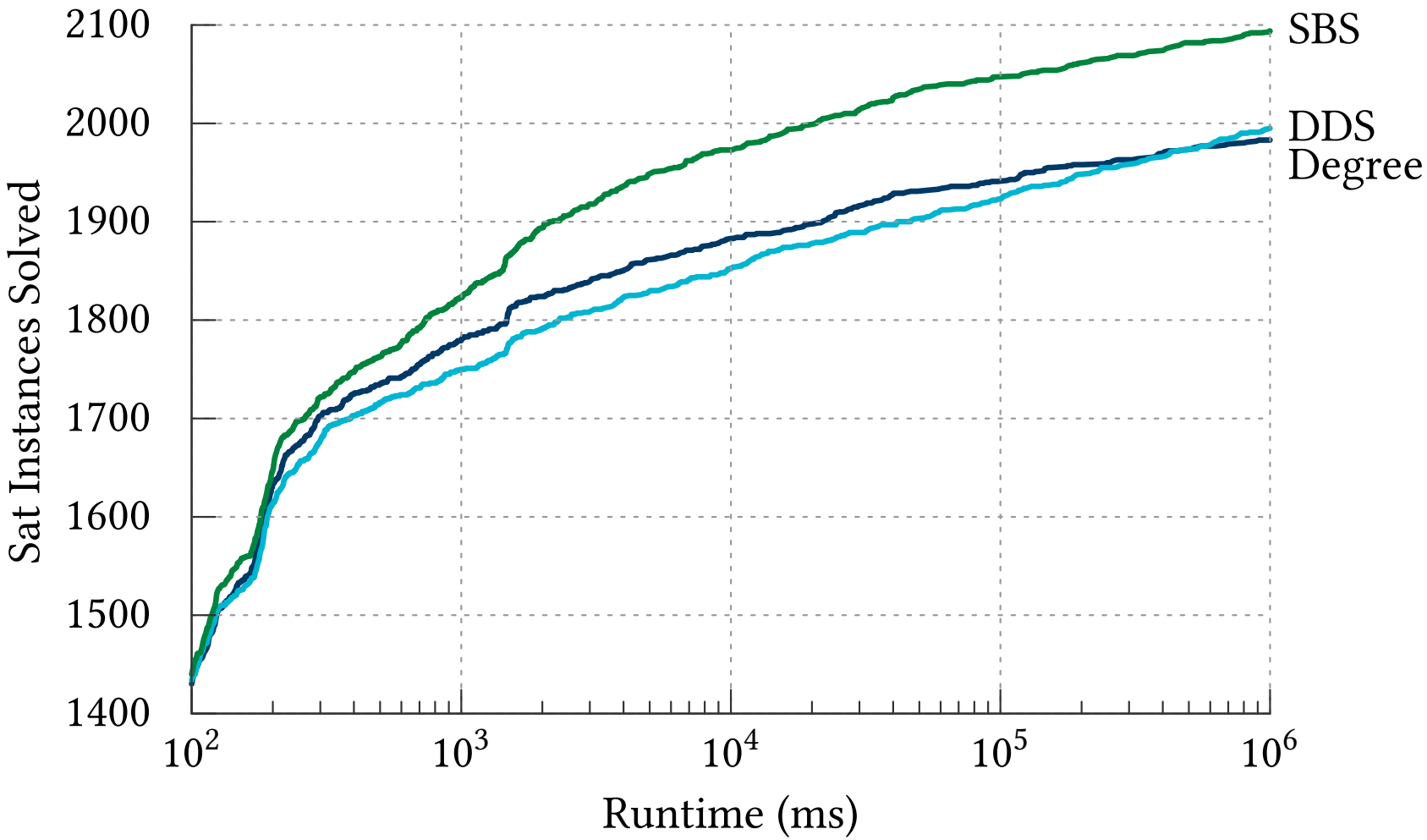
A Slightly Random Value-Ordering Heuristic



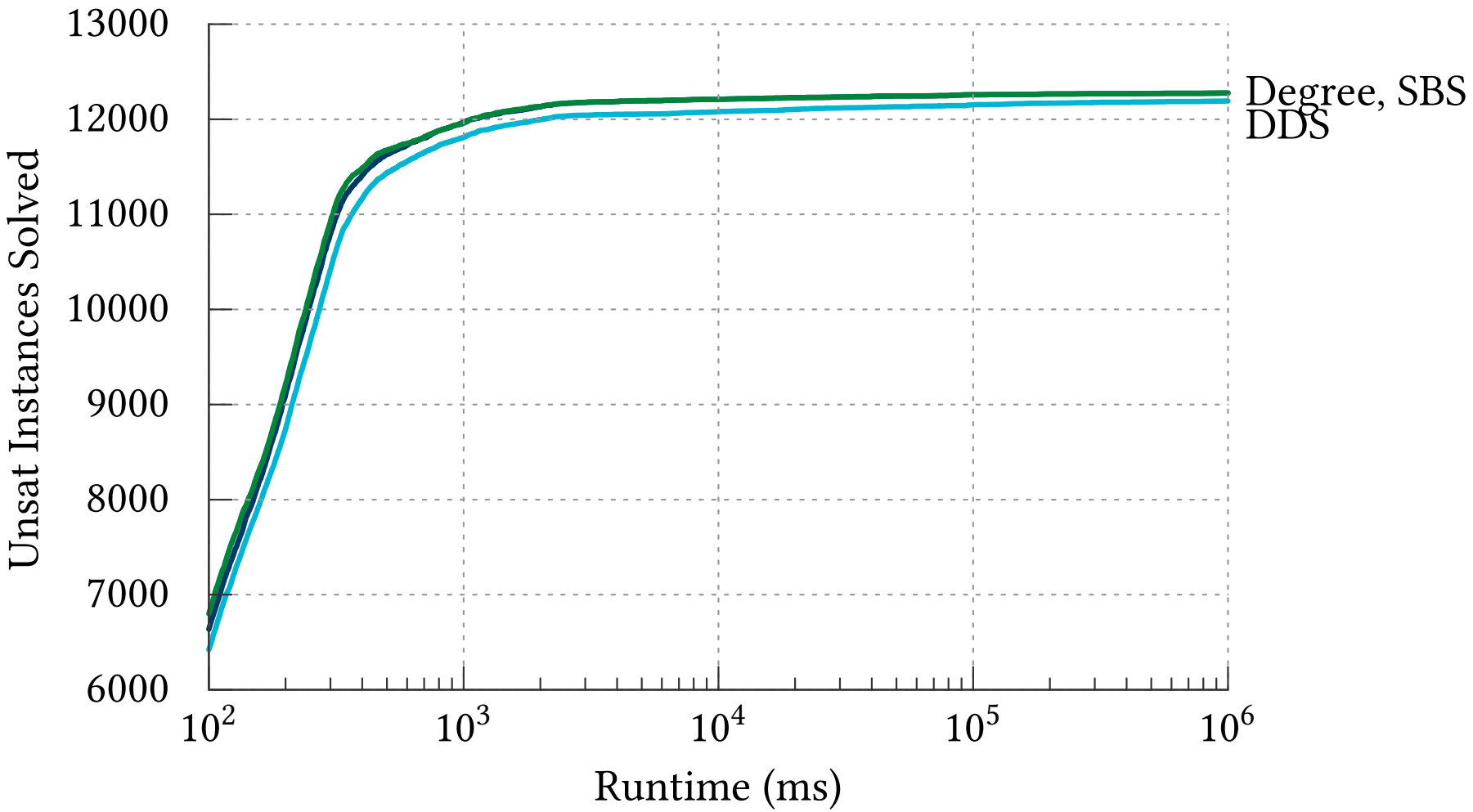
Restarts and Nogood Recording

- Aggressive restarts: every 100ms.
- Nogood recording and 2WL to avoid repeating work.

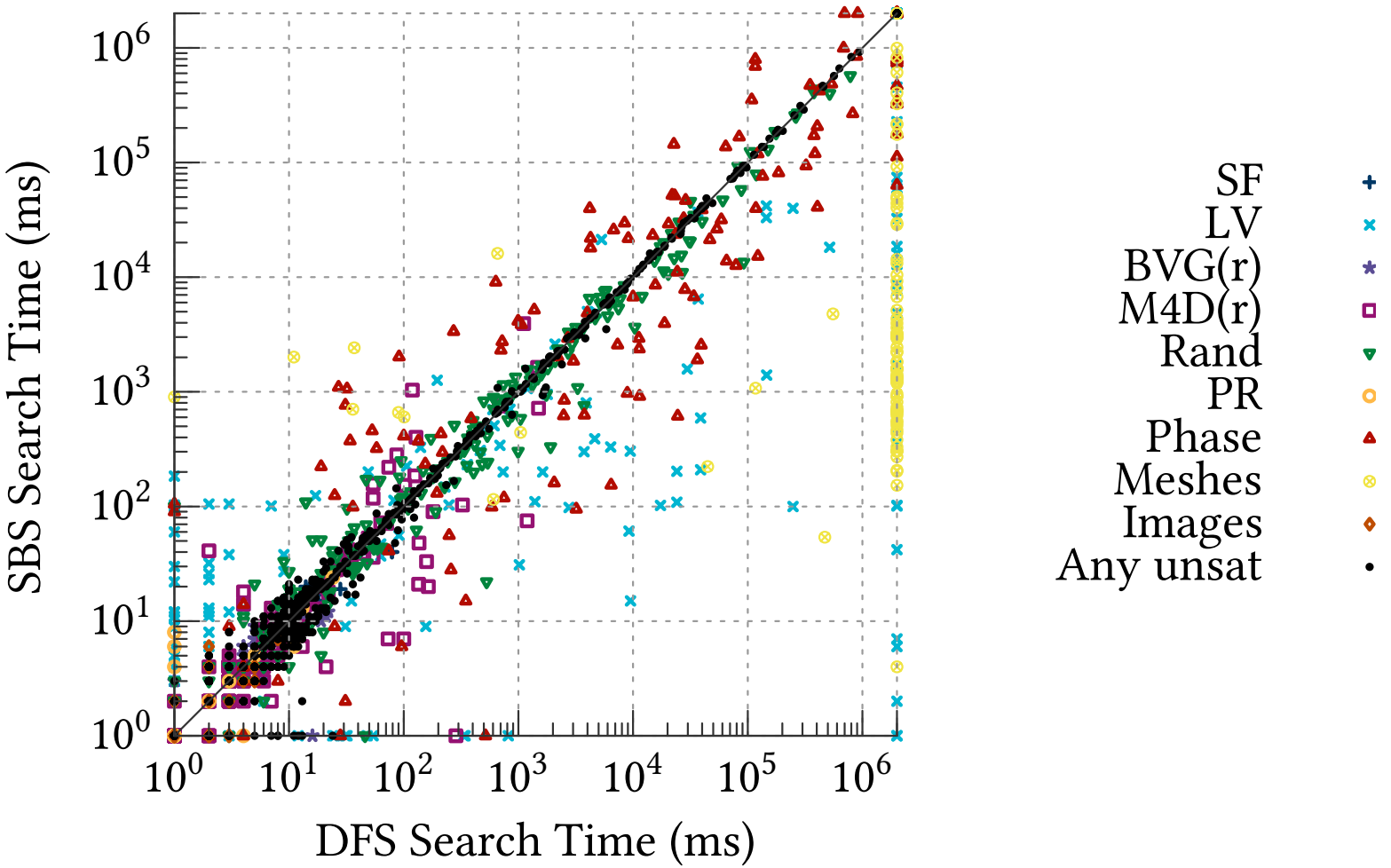
Is It Better?


2100
2000
1900
1800
1700
1600
1500
1400
Sat Instances Solved
10^2
10^3
10^4
10^5
10^6
Runtime (ms)
SBS
DDS
Degree

Is It Better?


13000
12000
11000
10000
9000
8000
7000
6000
Unsat Instances Solved
10^2
10^3
10^4
10^5
10^6
Runtime (ms)
Degree, SBS
DDS

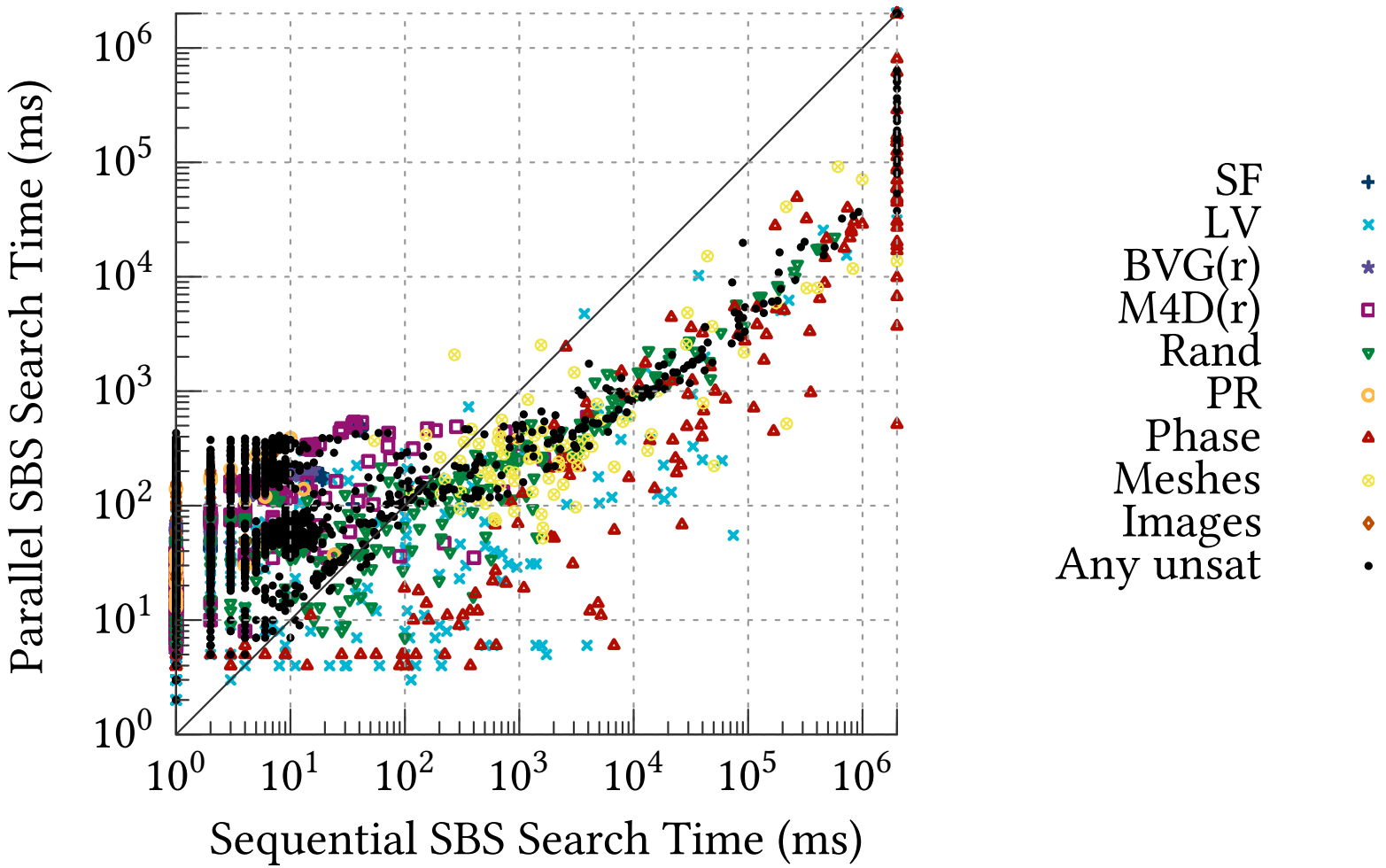
Is It Better?


SBS Search Time (ms)
DFS Search Time (ms)
10^0
10^1
10^2
10^3
10^4
10^5
10^6
SF
LV
BVG(r)
M4D(r)
Rand
PR
Phase
Meshes
Images
Any unsat

Parallel Search

- Each thread gets its own random seed.
- Barrier synchronise on restarts.
- Share nogoods.

Is It Even Betterer?



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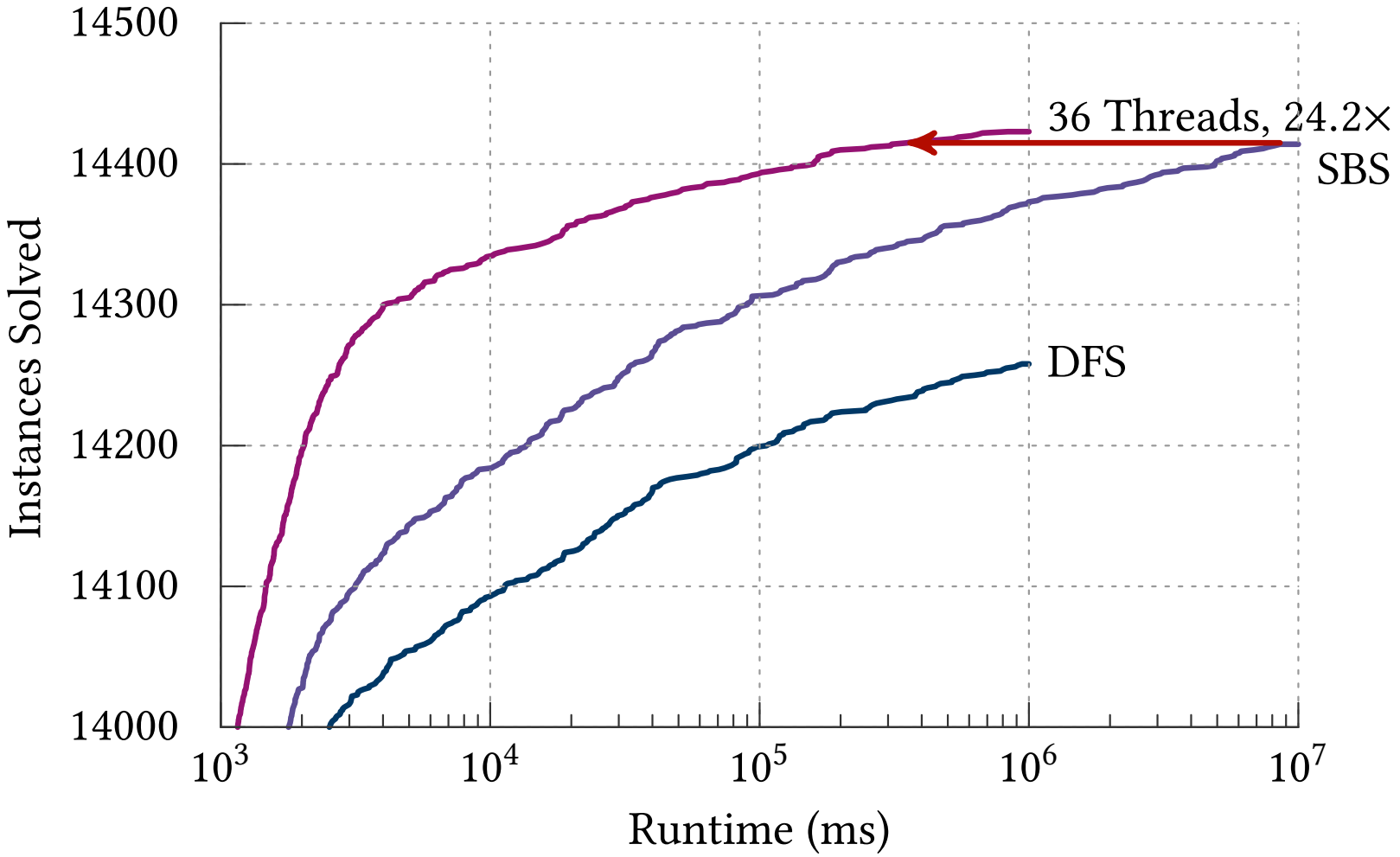

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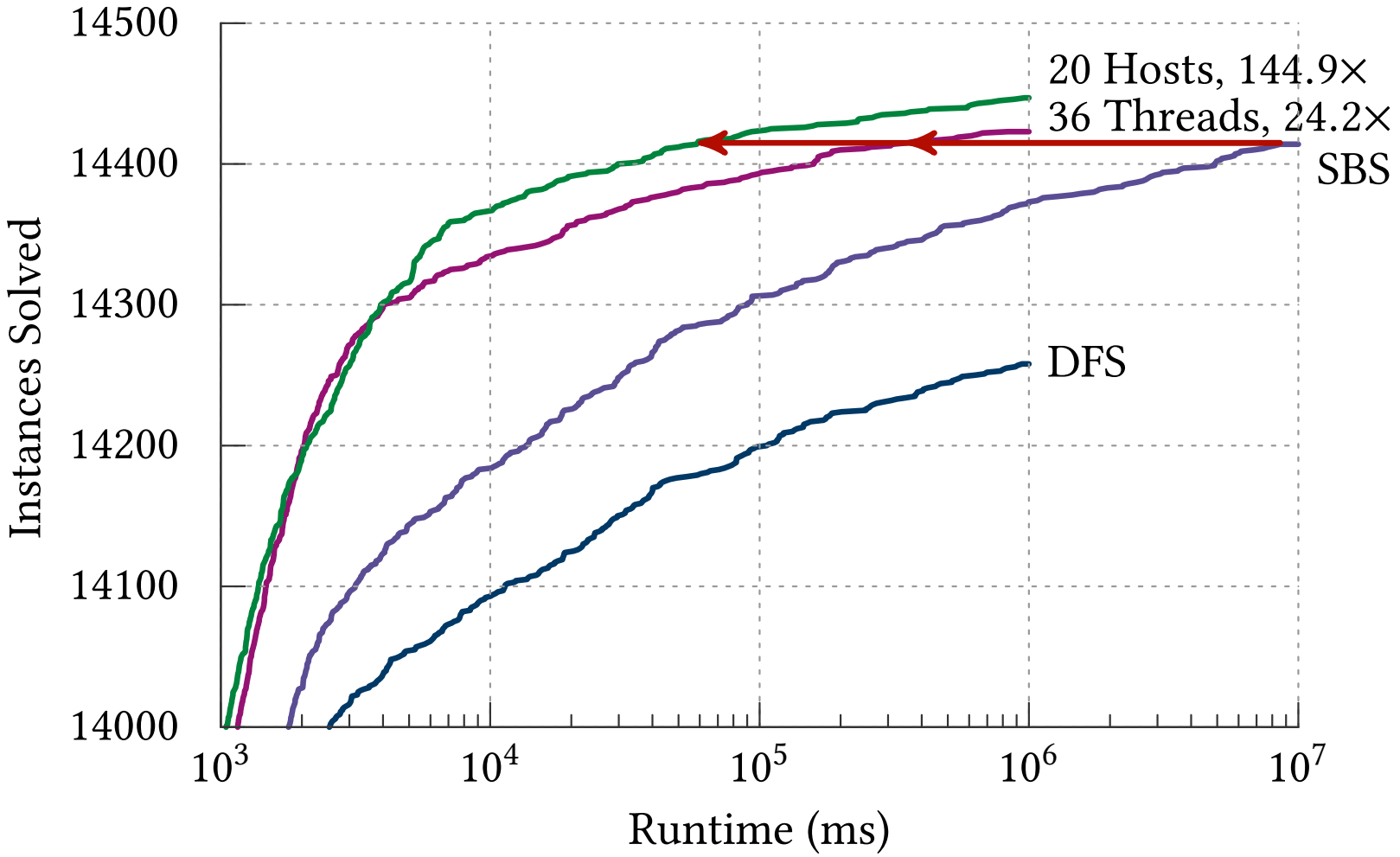

Broader Perspective

- Also good for maximum common subgraph algorithms.
- Does this work for CP in general?
- Can we finally kill off vanilla sequential backtracking search?

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